Disclaimer

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30.10.2017 **Advanced Quantum Field theory Exercise 3** Marvin Zanke

P4 (a)

$$\frac{1}{(2\pi)^{n/2}}\int\prod_{i=1}^{n}dx_i\,\exp\Big(-\tfrac{1}{2}\underbrace{x_k A_{kl} x_l}_{=x^T A x}+\underbrace{J_k x_k}_{=J^T x}\Big)$$

✓

"A real"? And what for? Pos def? ⇝ real values; pos. def ⇝ diagonalizable/ pos EV on diag. ⇝ $\sqrt{A}$ defined! Can diagonalise as A sym. ∇

$$\overset{A\text{ sym.}}{=}\frac{1}{(2\pi)^{n/2}}\int\prod_{i=1}^{n}dx_i\,\exp\Big(-\tfrac{1}{2}\underbrace{(x-A^{-1}J)^T A(x-A^{-1}J)}_{=x^TAx-x^TJ-J^Tx+J^TA^{-1}J}+\tfrac{1}{2}J^TA^{-1}J\Big)$$

$x^TJ = J^Tx$ (scalar^T=scalar)

$$\overset{\substack{\varphi:\,x\mapsto x+A^{-1}J\\ \det\varphi=1}}{=}\frac{1}{(2\pi)^{n/2}}\int\prod_{i=1}^{n}dx_i\,\exp\Big(-\tfrac{1}{2}x^TAx+\tfrac{1}{2}J^TA^{-1}J\Big)$$

✓

$l,k\leq n$? ⇝ yes

$$\overset{\substack{x\mapsto A^{-1/2}y\\ x^T\mapsto y^TA^{-1/2}}}{=}\frac{1}{(2\pi)^{n/2}}\exp\Big(\tfrac{1}{2}J^TA^{-1}J\Big)\int\prod_{i=1}^{n}dy_i\,\left|\det A^{-1/2}\right|\exp\Big(\overbrace{-\tfrac{1}{2}y^Ty}^{-\frac{1}{2}y_iy_i}\Big)$$

✓

$x\mapsto A^{-1/2}y$ or $x=A^{-1/2}y$? Both correct? ⇝ different notation

$$\overset{A\text{ pos. def.}}{=}\frac{1}{(2\pi)^{n/2}}(\det A)^{-1/2}\exp\Big(\tfrac{1}{2}J^TA^{-1}J\Big)\prod_{i=1}^{n}\int dy_i\,\exp\Big(-\tfrac{1}{2}y_i^2\Big)$$

$$=\frac{1}{(2\pi)^{n/2}}(\det A)^{-1/2}\exp\Big(\tfrac{1}{2}J^TA^{-1}J\Big)\prod_{i=1}^{n}\sqrt{2\pi}$$

$$=(\det A)^{-1/2}\exp\Big(\tfrac{1}{2}J^TA^{-1}J\Big)=(\det A)^{-1/2}\exp\Big(\tfrac{1}{2}J_kA^{-1}_{kl}J_l\Big)$$

(b)

$$\frac{1}{(2\pi i)^n}\int\prod_{i=1}^{n}dz_i^*dz_i\,\exp\Big(-\underbrace{z_k^*H_{kl}z_l}_{z^\dagger Hz}+\underbrace{J_k^*z_k}_{J^\dagger z}+\underbrace{J_kz_k^*}_{J^Tz^*}\Big)$$

$$=\frac{1}{(2\pi i)^n}\int\prod_{i=1}^{n}dz_i^*dz_i\,\exp\Big(-\underbrace{(z-H^{-1}J)^\dagger H(z-H^{-1}J)}_{=z^\dagger Hz-z^\dagger J-J^\dagger z+J^\dagger H^{-1}J}+J^\dagger H^{-1}J\Big)$$

$z^\dagger J = J^Tz^*$ (scalar^T=scalar)

⇝ $dz_i^*=dw_i^*$?

$$\overset{\substack{\varphi:\,z\mapsto z+H^{-1}J\\ \det\varphi=1}}{=}\frac{1}{(2\pi i)^n}\int\prod_{i=1}^{n}dz_i^*dz_i\,\exp\Big(-z^\dagger Hz+J^\dagger H^{-1}J\Big)$$

$$\overset{\substack{z=H^{-1/2}w\\ z^\dagger=w^\dagger H^{-1/2}}}{=}\frac{1}{(2\pi i)^n}\exp\Big(J^\dagger H^{-1}J\Big)\int\prod_{i=1}^{n}dw_i^*dw_i\,\exp\Big(\overbrace{-w^\dagger w}^{-w_i^*w_i}\Big)\left|\det H^{-1/2}\right|\left|\det H^{-1/2}\right|$$

Can transform in each w_i, w_i^* seperately?

$w=x+iy$, $w^*=x-iy$ ⇝ $w_i=x_i+iy_i$, $w_i^*=x_i-iy_i$

$$\begin{pmatrix}w_i\\ w_i^*\end{pmatrix}=\underbrace{\begin{pmatrix}1 & i\\ 1 & -i\end{pmatrix}}_{T}\begin{pmatrix}x_i\\ y_i\end{pmatrix}\ \leadsto\ |\det T|=2i$$

$$= \frac{(\det H)^{-1}}{\pi^n} \exp(J^\dagger H^{-1} J) \int \prod_{i=1}^{n} dx_i dy_i \exp(-(x^T - i y^T)(x + i y))$$

$$= \frac{(\det H)^{-1}}{\pi^n} \exp(J^\dagger H^{-1} J) \int \prod_{i=1}^{n} dx_i dy_i \exp\left(-x^T x - y^T y + \underbrace{i y^T x}_{i x^T y} - i x^T y\right)$$

$$= \frac{(\det H)^{-1}}{\pi^n} \exp(J^\dagger H^{-1} J) \int \prod_{i=1}^{n} dx_i dy_i \exp(-\underbrace{x^T x}_{x_i x_i}) \exp(-\underbrace{y^T y}_{y_i y_i})$$

$$= \frac{(\det H)^{-1}}{\pi^n} \exp(J^\dagger H^{-1} J) \prod_{i=1}^{n} \int dx_i \exp(-x_i^2) \int dy_i \exp(-y_i^2)$$

$$= \frac{(\det H)^{-1}}{\pi^n} \exp(J^\dagger H^{-1} J) \prod_{i=1}^{n} \sqrt{\pi} \sqrt{\pi}$$

$$= (\det H)^{-1} \exp(J^\dagger H^{-1} J) = (\det H)^{-1} \exp(J_k^* H_{kl} J_l)$$

(c) $(\tilde{\xi}_1, \tilde{\xi}_2, \ldots \tilde{\xi}_{2n}) \equiv (\xi_1^*, \xi_2^*, \ldots, \xi_n^*, \xi_1, \xi_2, \ldots, \xi_n)$

$(\tilde{\eta}_1, \tilde{\eta}_2, \ldots, \tilde{\eta}_{2n}) = (\eta_1^*, \eta_2^*, \ldots, \eta_n^*, \eta_1, \eta_2, \ldots, \eta_n)$

$\tilde{\xi}_i = M_{ij}\tilde{\eta}_j$

Want to show: $\int d\xi_1^* d\xi_1 \ldots d\xi_n^* d\xi_n \, F(\xi^*, \xi) = \left|\frac{\partial(\eta^*, \eta)}{\partial(\xi^*, \xi)}\right| \int d\eta_1^* d\eta_1 \ldots d\eta_n^* d\eta_n \, F(\xi^*(\eta^*, \eta), \xi(\eta^*, \eta))$

Basis given by ξ_i and ξ_i^*? ⇝ yes, all independent, regular Grassmann vars!

The generators for the first set are $\tilde{\xi}_1, \ldots, \tilde{\xi}_{2n}$ and for the second set $\tilde{\eta}_1, \ldots, \tilde{\eta}_{2n}$

How to show decomposition? ⇝ Just use $\xi_i^2 = 0$ in order to show the exact decomp. General decomp. given per def. but monomials: $\theta_i\theta_j$, etc

We first denote that $F(\tilde{\xi}) = F^{(0)} + \sum_i F_i^{(1)}\tilde{\xi}_i + \ldots + \sum_{i \ldots k} F_{i \ldots k}^{(2n)} \tilde{\xi}_i \ldots \tilde{\xi}_k$

and $F(\tilde{\eta}) = \tilde{F}^{(0)} + \sum_i \tilde{F}_i^{(1)} \tilde{\eta}_i + \ldots + \sum_{i \ldots k} \tilde{F}_{i \ldots k}^{(2n)} \tilde{\eta}_i \ldots \tilde{\eta}_k$

$\Longrightarrow \int d\tilde{\xi}_1 \ldots d\tilde{\xi}_{2n} \, F(\tilde{\xi}) = \int d\tilde{\xi}_1 \ldots d\tilde{\xi}_{2n} \left(\sum_{i \ldots k} F_{i \ldots k}^{(2n)} \tilde{\xi}_i \ldots \tilde{\xi}_k\right)$

only terms to survive after integration

$= (2n)! \, F_{1 \ldots 2n}^{(2n)}$

$= (2n)! \, F_{1 \ldots 2n}$ because $F_{i \ldots k}^{(2n)}$ antisym?

and $\int d\tilde{\eta}_1 \ldots d\tilde{\eta}_{2n} \, F(\tilde{\eta}) = \int d\tilde{\eta}_1 \ldots d\tilde{\eta}_{2n} \left(\sum_{i \ldots k} \tilde{F}_{i \ldots k}^{(2n)} \tilde{\eta}_i \ldots \tilde{\eta}_k\right)$

$= (2n)! \, \tilde{F}_{1 \ldots 2n}^{(2n)}$

Nicer way from LHS to RHS? ⇝ see tutorial

$$\leadsto \frac{\int d\tilde{\xi}_1 \ldots d\tilde{\xi}_{2n} \, F(\tilde{\xi})}{\int d\tilde{\eta}_1 \ldots d\tilde{\eta}_{2n} \, F(\tilde{\eta})} = \frac{F_{1 \ldots 2n}^{(2n)}}{\tilde{F}_{1 \ldots 2n}^{(2n)}} \quad (*)$$

Start from $F_{1 \ldots 2n}^{(2n)} \tilde{\xi}_1 \ldots \tilde{\xi}_{2n} = F_{1 \ldots 2n}^{(2n)} M_{1i}\tilde{\eta}_i \ldots M_{2nk}\tilde{\eta}_k$

$= F_{1 \ldots 2n}^{(2n)} M_{1i} \ldots M_{2nk} \, \epsilon_{i \ldots k} \, \tilde{\eta}_1 \ldots \tilde{\eta}_{2n}$

$\stackrel{\text{per def. of det.}}{=} F_{1 \ldots 2n}^{(2n)} \det M \, \tilde{\eta}_1 \ldots \tilde{\eta}_{2n} \stackrel{!}{=} \tilde{F}_{1 \ldots 2n}^{(2n)} \tilde{\eta}_1 \ldots \tilde{\eta}_{2n}$

$\Longrightarrow \tilde{F}_{1 \ldots 2n}^{(2n)} = \det M \, F_{1 \ldots 2n}^{(2n)}$

$\Longrightarrow (*) = (\det M)^{-1} = \frac{\partial(\tilde{\eta})}{\partial(\tilde{\xi})} = \frac{\partial(\eta^*, \eta)}{\partial(\xi^*, \xi)}$

Where from $|\ldots|$ (mod.)? ⇝ No mod but determinant!

(d)

$$\int \prod_{i=1}^{n} d\eta_i^* d\eta_i \exp(\underbrace{-\eta_k^* A_{kl} \eta_l}_{\eta^+ H \eta} + \underbrace{\xi_k^* \eta_k}_{\xi^+ \eta} + \underbrace{\xi_k \eta_k^*}_{\xi^T \eta^*})$$

Indep. Grassmann vars. commute? ⇝ No, anti-commute!

$$= \int \prod_{i=1}^{n} d\eta_i^* d\eta_i \exp(-\underbrace{(\eta - H^{-1}\xi)^+ H (\eta - H^{-1}\xi)}_{= \eta^+ H \eta - \underbrace{\eta^+ \xi}_{\xi^T \eta^*} - \xi^+ \eta + \xi^+ H^{-1} \xi} + \xi^+ H^{-1} \xi)$$

$$\overset{\eta \mapsto \eta + H^{-1}\xi}{=} \int \prod_{i=1}^{n} d\eta_i^* d\eta_i \exp(-\eta^+ H \eta + \xi^+ H^{-1} \xi)$$

$$\overset{k = H^{1/2}\eta}{=} \left\{ \int \prod_{i=1}^{n} dk_i^* dk_i \left| \frac{\partial(k^*, k)}{\partial(\eta^*, \eta)} \right| \exp(-k^+ k) \right\} \exp(\xi^+ H^{-1} \xi)$$

Zweifache Subst. oder $\frac{\partial(\cdot,\cdot)}{\partial(\cdot,\cdot)} = \frac{\partial(\cdot)}{\partial(\cdot)} \frac{\partial(\cdot)}{\partial(\cdot)}$? ⇝ As we transform independently, we get $\begin{pmatrix} A & \\ 0 & B \end{pmatrix}$ ⇝ split

$$= \left\{ \frac{\partial(k^*)}{\partial(\eta^*)} \frac{\partial(k)}{\partial(\eta)} \int \prod_{i=1}^{n} dk_i^* dk_i \exp(\underbrace{-k^+ k}_{-k_i^* k_i}) \right\} \exp(\xi^+ H^{-1} \xi)$$

$$= (\det H) \left\{ \int \prod_{i=1}^{n} dk_i^* dk_i \left(\sum_{i=0}^{\infty} \frac{(-k_j^* k_j)^i}{i!} \right) \right\} \exp(\xi^+ H^{-1} \xi)$$

$\{k_i, k_j^*\} = 0$? ⇝ yes, ind. variables

$$\overset{\text{only int. w/ 2n G.V. survives}}{=} (\det H) \left\{ \int dk_1^* dk_1 \dots dk_n^* dk_n (\underbrace{-k_j^* k_j}_{k_j k_j^*})^n \right\} \exp(\xi^+ H^{-1} \xi)$$

$k_i = \phi_i + i\chi_i$ as well? ⇝ k_i^* just different Grassmann var.

$$= (\det H) \exp(\xi^+ H^{-1} \xi) = (\det H) \exp(\xi_k^* H^{-1}_{kl} \xi_l)$$

Rather $|\det H|$ than $\det H$? ⇝ $\left|\frac{\partial(\cdot)}{\partial(\cdot)}\right|$ no modulo but det. ✓

P5) $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$

Only for scalar field or other too that?

(a)

Where from gen. fct.? ⇝ see next page from tutorial

$$W[j] = \frac{\exp\left(-i\frac{\lambda}{4!}\int d^4z \left(\frac{1}{i}\frac{\delta}{\delta j(z)}\right)^4\right)\exp\left(-\frac{i}{2}\int d^4x\, d^4y\, j(x) D_F(x-y) j(y)\right)}{\left\{\exp(\ldots)\exp(\ldots)\right\}\Big|_{j=0}} = \frac{Z[j]}{Z[0]}$$

We first take a look at the numerator (later simply setting $j=0$ yields the denominator), expanding in λ up $\mathcal{O}(\lambda^1)$ for the first term:

$$Z[j] = \left\{1 - i\frac{\lambda}{4!}\int d^4z\left(\frac{1}{i}\frac{\delta}{\delta j(z)}\right)^4 + \mathcal{O}(\lambda^2)\right\}\exp\left(-\frac{i}{2}\int d^4x\, d^4y\, j(x) D_F(x-y) j(y)\right)$$

Using shorthand notation: $j_x \equiv j(x)$, $D_{xy} = D_F(x-y)$

$j_x D_{xy} j_y \equiv \int d^4x\, d^4y\, j(x) D_F(x-y) j(y)$ (in general, integrating over indices appearing twice, except for z)

$\therefore (j_x D_{xz})^2 = j_x D_{xz} j_y D_{yz} \equiv \int d^4x\, j(x) D_F(x-z) \int d^4y\, j(y) D_F(y-z)$

Not integrating over z in this notation? e.g. $j_x D_{xz} j_y D_{yz}$

$$= \left\{1 - i\frac{\lambda}{4!}\int d^4z\left(\frac{\delta}{\delta j_z}\right)^4 + \mathcal{O}(\lambda^2)\right\}\exp\left(-\frac{i}{2} j_x D_{xy} j_y\right)$$

Now, we calculate

$$\frac{\delta}{\delta j_z}\frac{\delta}{\delta j_z}\frac{\delta}{\delta j_z}\frac{\delta}{\delta j_z}\exp\left(-\frac{i}{2} j_x D_{xy} j_y\right)$$

$$= \frac{\delta}{\delta j_z}\frac{\delta}{\delta j_z}\frac{\delta}{\delta j_z}\left\{-\frac{i}{2} D_{zy} j_y - \frac{i}{2} j_x D_{xz}\right\}\exp\left(-\frac{i}{2} j_x D_{xy} j_y\right)$$

change int. vars. & D_F sym.; $j(x)$ class. current

$$= \frac{\delta}{\delta j_z}\frac{\delta}{\delta j_z}\frac{\delta}{\delta j_z}\left\{-i\, j_x D_{xz}\right\}\exp\left(-\frac{i}{2} j_x D_{xy} j_y\right)$$

$j(x)$ class. → commutes? ⇝ if not said differently, classical field and can be pulled to the front for bosons. For fermions, $j(x)$ is a Grassmann var as well ⇝ (−) sign for exchange

$$= \frac{\delta}{\delta j_z}\frac{\delta}{\delta j_z}\left\{-i D_{zz} - j_x D_{xz} j_y D_{yz}\right\}\exp\left(-\frac{i}{2} j_x D_{xy} j_y\right)$$

$$= \frac{\delta}{\delta j_z}\frac{\delta}{\delta j_z}\left\{-i D_{zz} - (j_x D_{xz})^2\right\}\exp\left(-\frac{i}{2} j_x D_{xy} j_y\right)$$

$$= \frac{\delta}{\delta j_z}\left\{-D_{zz} j_x D_{xz} - 2(j_x D_{xz}) D_{zz} + i(j_x D_{xz})^3\right\}\exp\left(-\frac{i}{2} j_x D_{xy} j_y\right)$$

Terms like $(j_x D_{xz})^2$ stand for various indices? ⇝ Yes

$$= \left\{-D_{zz}D_{zz} + iD_{zz}(j_x D_{xz})^2 - 2D_{zz}D_{zz} + 2iD_{zz}(j_x D_{xz})^2 + 3i(j_x D_{xz})^2 D_{zz} + (j_x D_{xz})^4\right\}\exp\left(-\frac{i}{2} j_x D_{xy} j_y\right)$$

$$= \left\{ -3 D_{zz} D_{zz} + 6i D_{zz} (j_x D_{xz})^2 + (j_x D_{xz})^4 \right\} \exp\left(-\tfrac{i}{2} j_x D_{xy} j_y\right)$$

$$\rightsquigarrow Z[j] = \Big\{ 1 - i \frac{\lambda}{4!} \int d^4z \big(-3 D_F(0) D_F(0) + 6i D_F(0) (j_x D_{xz})(j_y D_{yz})$$
$$+ (j_x D_{xz})(j_y D_{yz})(j_k D_{kz})(j_l D_{lz}) \big) + \mathcal{O}(\lambda^2) \Big\}$$
$$\times \exp\left(-\tfrac{i}{2} j_x D_{xy} j_y\right)$$

$$\rightsquigarrow Z[0] = \Big\{ 1 - i \frac{\lambda}{4!} \int d^4z \big(-3 D_F(0) D_F(0) \big) + \mathcal{O}(\lambda^2) \Big\}$$

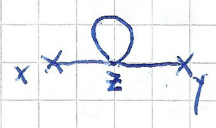
Taylor again?

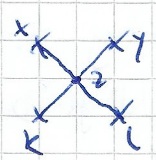
We now have a form like $\dfrac{1 - \lambda (x+y+z)}{1 - \lambda x} = 1 - \lambda (y+z) + \mathcal{O}(\lambda^2)$
for $W[j]$

X already scattering tree level?

$$\rightsquigarrow W[j] = \Big\{ 1 - i \frac{\lambda}{4!} \int d^4z \big(6i D_F(0) (j_x D_{xz})(j_y D_{yz}) + (j_x D_{xz})(j_y D_{yz})(j_k D_{kz})(j_l D_{lz}) \big)$$
$$+ \mathcal{O}(\lambda^2) \Big\} \exp\left(-\tfrac{i}{2} j_x D_{xy} j_y\right)$$

What about exp factor?





Why the "1"?

Why $D(0) = \underset{z}{8}$?
No z in it no more
but integral over z?

(b) We now want to calculate the 4-point function

$$\langle 0|T\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)|0\rangle = \frac{\delta^4 W[j]}{\delta j(x_1)\delta j(x_2)\delta j(x_3)\delta j(x_4)}\bigg|_{j=0}$$

Using the result of (a), we will now calculate this derivative, where we again use our shorthand notation and already use the $\{\ldots\}|_{j=0}$ in the sense, that we instantantly neglect terms which would yield zero after setting $j=0$ in the end.

$$\frac{\delta^4 W[j]}{\delta j_{x_1}\delta j_{x_2}\delta j_{x_3}\delta j_{x_4}}\bigg|_{j=0} = \frac{\delta}{\delta j_{x_1}}\frac{\delta}{\delta j_{x_2}}\frac{\delta}{\delta j_{x_3}}\frac{\delta}{\delta j_{x_4}}\Big\{1 - i\frac{\lambda}{4!}\int d^4z\,\big(6iD_F(0)(j_x D_{xz})^2 + (j_x D_{xz})^4 + \mathcal{O}(\lambda^2)\big\}\exp(-\tfrac{i}{2}j_x D_{xy} j_y)\Big)\Big|_{j=0}$$

$\int d^4z$ whenever a z occurs!

explicitly write d^4z integration here? → has to be written out or introduce new notation w/ terms (bar) if no z integration

$$= \frac{\delta}{\delta j_{x_1}}\frac{\delta}{\delta j_{x_2}}\frac{\delta}{\delta j_{x_3}}\Big\{-i j_x D_{xx_4} + \frac{\lambda}{4}D_F(0)\cdot 2(j_x D_{xz})D_{x_4z} - i\frac{\lambda}{4}D_F(0)(j_x D_{xz})^2(j_y D_{yx_4}) - i\frac{\lambda}{6}(j_x D_{xz})^3 D_{x_4z} \underbrace{-\frac{\lambda}{4!}(j_x D_{xz})^4(j_y D_{yx_4})}\Big\}\exp(-\tfrac{i}{2}j_x D_{xy} j_y)\Big|_{j=0}$$

Can already be neglected, as it has $5\times j_x$, while theres only 3 derivatives left and $j=0$ in the end!

again, neglecting terms with $j^x, x>2$

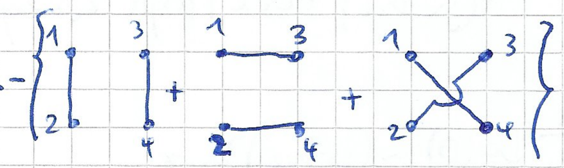
$$= \frac{\delta}{\delta j_{x_1}}\frac{\delta}{\delta j_{x_2}}\Big\{-iD_{x_3x_4} - j_x D_{xx_4} j_y D_{yx_3} + \frac{\lambda}{2}D_F(0)D_{x_3z}D_{x_4z} - i\frac{\lambda}{2}D_F(0)(j_x D_{xz})D_{x_4z}(j_y D_{yx_3}) - i\frac{\lambda}{2}D_F(0)(j_x D_{xz})D_{x_3z}(j_y D_{yx_4}) - i\frac{\lambda}{4}D_F(0)(j_x D_{xz})^2 D_{x_3x_4} - i\frac{\lambda}{2}(j_x D_{xz})^2 D_{x_3z}D_{x_4z}\Big\}\exp(-\tfrac{i}{2}j_x D_{xy} j_y)\Big|_{j=0}$$

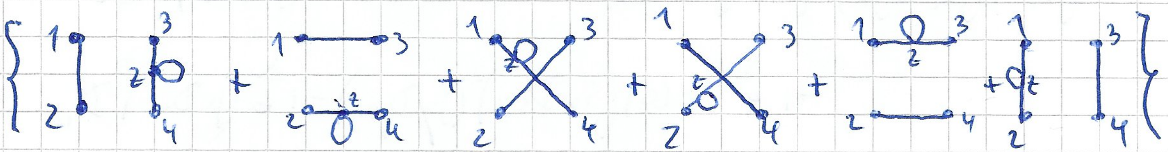
$$= \frac{\delta}{\delta j_{x_1}}\Big\{-D_{x_3x_4}(j_x D_{xx_2}) - D_{x_2x_4} j_y D_{yx_3} - j_x D_{xx_4} D_{x_2x_3} - i\frac{\lambda}{2}D_F(0)D_{x_3z}D_{x_4z}(j_x D_{xx_2}) - i\frac{\lambda}{2}D_F(0)D_{x_2z}D_{x_4z}(j_y D_{yx_3}) - i\frac{\lambda}{2}D_F(0)(j_x D_{xz})D_{x_4z}D_{x_2x_3} - i\frac{\lambda}{2}D_F(0)D_{x_2z}D_{x_3z}(j_y D_{yx_4}) - i\frac{\lambda}{2}D_F(0)(j_x D_{xz})D_{x_2z}D_{x_3x_4} - i\frac{\lambda}{2}D_F(0)(j_x D_{xz})D_{x_3z}D_{x_2x_4} - i\lambda(j_x D_{xz})D_{x_2z}D_{x_3z}D_{x_4z}\Big\}\exp(\ldots)$$

$$= \Big\{ -D_{x_3x_4}D_{x_1x_2} - D_{x_2x_4}D_{x_1x_3} - D_{x_1x_4}D_{x_2x_3} - i\frac{\lambda}{2}\Big(D_F(0)D_{x_3z}D_{x_4z}D_{x_1x_2}$$
$$+ D_F(0)D_{x_2z}D_{x_4z}D_{x_1x_3} + D_F(0)D_{x_1z}D_{x_4z}D_{x_2x_3} + D_F(0)D_{x_2z}D_{x_3z}D_{x_1x_4}$$
$$+ D_F(0)D_{x_1z}D_{x_3z}D_{x_2x_4} + D_F(0)D_{x_1z}D_{x_2z}D_{x_3x_4}\Big) - i\lambda D_{x_1z}D_{x_2z}D_{x_3z}D_{x_4z}\Big\}$$
$$\times \exp\left(-\frac{i}{2}j_x D_{xy} j_y\right)\Big|_{j=0}$$

$$= -\Big\{ D_{x_1x_2}D_{x_3x_4} + D_{x_1x_3}D_{x_2x_4} + D_{x_1x_4}D_{x_2x_3}\Big\}$$
$$-i\frac{\lambda}{2}\Big\{ D_F(0)D_{x_1x_2}D_{x_3z}D_{x_4z} + D_F(0)D_{x_1x_3}D_{x_2z}D_{x_4z} + D_F(0)D_{x_2x_3}D_{x_1z}D_{x_4z}$$
$$+ D_F(0)D_{x_1x_4}D_{x_2z}D_{x_3z} + D_F(0)D_{x_2x_4}D_{x_1z}D_{x_3z} + D_F(0)D_{x_3x_4}D_{x_1z}D_{x_2z}\Big\}$$
$$-i\lambda D_{x_1z}D_{x_2z}D_{x_3z}D_{x_4z}$$

No ext currents?

$\rightsquigarrow -$ 

$-i\frac{\lambda}{2}$ 

Not clear, where $D_F(0)$ is? Clear because of z-integration?

$-i\lambda$