13.11.2017 / Advanced Quantum Field theory Ex. sheet 5

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Where in Feynman rules is Photon prop.? P7)
$\epsilon_\mu$ even after ortg. antip.?
⇝ doesn't matter

$$\Gamma^\mu(p',p) = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}(p'-p)_\nu}{2m} F_2(q^2), \quad i\sigma^{\mu\nu} = -\frac{1}{2}[\gamma^\mu,\gamma^\nu]$$

Scalars: always $k_\mu q^\mu \neq 0$ or
Why is $C=0$?
But $B=0$ as well as $q_\mu\Gamma^\mu=0$ has to hold?
⇝ $q_\mu(p^\mu+p'^\mu)=0$ and $q_\mu\gamma^\mu=0$ w/ $\bar{u}$ … $u$ always ⇝ only $C=0$ needed ∇

$$P_2(p',p)^\mu = (\not p + m)\Delta_2^\mu(p',p)(\not p' + m)$$

$$\Delta_2^\mu(p',p) = \underbrace{-\frac{2m^2}{(d-2)q^2(q^2-4m^2)}}_{\equiv C}\left(\gamma^\mu + \underbrace{\frac{(d-2)q^2+4m^2}{q^2-4m^2}\cdot\frac{(p^\mu+p'^\mu)}{2m}}_{\equiv k^\mu}\right)$$

$\Gamma^\mu$ as vector ✓
→ lin. comb of vector obj. in above expr.
$\gamma^\mu, p^\mu, p'^\mu$?
⇝ only possibilities; can be seen by $N(U(N))$ trafo and ✓ $\langle p'|J^\mu|p\rangle$

We further have $iM = \bar{u}(p,s')\Gamma^\mu(p',p)u(p,s)\epsilon_\mu(q,r)(-ie\mu^{2-d/2})$

$$\Gamma^\mu(p',p) = -ie^2\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{\gamma^\nu(\not p'-\not k+m)\gamma^\mu(\not p-\not k+m)\gamma_\nu}{[(p'-k)^2-m^2+i\epsilon][(p-k)^2-m^2+i\epsilon][k^2+i\epsilon]}$$

Ward id. valid w/o $\bar{u}$ and $u$?
⇝ should ~~also~~ not hold w/o those

and $p+q=p' \rightsquigarrow \underline{q=p'-p}$

$$\rightsquigarrow q^2 = 2m^2 - 2pp' \iff \underline{p\cdot p' = m^2 - \frac{q^2}{2}}$$

Absorb add. factors from Gordon id. in $A(q^2)$?

We will encounter terms like

- $\gamma_\mu\gamma^\mu = d$ (given)
- $\gamma^\mu\gamma^\lambda\gamma_\mu = (2g^{\mu\lambda}-\gamma^\lambda\gamma^\mu)\gamma_\mu = -d\gamma^\lambda + 2\gamma^\lambda = (2-d)\gamma^\lambda$
- $Tr(\gamma_\nu\gamma_\mu) = tr(2g_{\mu\nu}-\gamma_\mu\gamma_\nu)$
  $\iff Tr(\gamma_\nu\gamma_\mu) = 4g_{\mu\nu}$

Where from P? ✓
⇝ construct from orthogonal subspace ✓

- $Tr(\gamma^\mu\gamma_\mu) = 4d$
- $Tr(\gamma_\kappa\gamma_\lambda\gamma_\mu\gamma_\nu) = 4(g_{\kappa\lambda}g_{\mu\nu} + g_{\kappa\nu}g_{\lambda\mu} - g_{\mu\kappa}g_{\lambda\nu})$
- $\gamma^\mu\gamma_\kappa\gamma_\nu\gamma_\mu = \gamma^\mu\gamma_\kappa(2g_{\mu\nu}-\gamma_\mu\gamma_\nu) = -(2-d)\gamma_\kappa\gamma_\nu + 2\gamma_\nu\gamma_\kappa$
- $Tr(\gamma^\mu\gamma_\kappa\gamma_\nu\gamma_\mu) = (d-2)Tr(\gamma_\kappa\gamma_\nu) + 2Tr(\gamma_\nu\gamma_\kappa) = d\,Tr(\gamma_\kappa\gamma_\nu)$

$g^\mu_\mu = d$ but $\gamma^\mu$ 4x4 matrices?
⇝ Actually it's $D g^{\mu\nu} = tr(\gamma^\mu\gamma^\nu)$ but those D's cancel out after regularisation (counter-terms) and that's why $D=4$ is ok∇ d

We will now show that $Tr[P_2^\mu(p',p)\Gamma_\mu(p',p)] = F_2(q^2)$ for a general $\Gamma_\mu(p',p)$.

$$Tr[P_2^\mu(p',p)\Gamma_\mu(p',p)] = Tr\left[(\not p+m)\{C(\gamma^\mu+k^\mu)\}(\not p'+m)\left\{\gamma_\mu F_1(q^2) - \frac{\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu}{4m}(p'-p)^\nu F_2(q^2)\right.\right.$$

$$= C\,Tr\left[(\not p+m)(\gamma^\mu+k^\mu)(\not p'+m)\left\{\gamma_\mu F_1(q^2) - \frac{(p'-p)^\nu}{4m}F_2(q^2)(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)\right\}\right]$$

#odd vanish

$$= C\,\mathrm{Tr}\left[\left(\not{p}\gamma^\mu \not{p}' + k^\mu m \not{p} + k^\mu m \not{p}' + m^2 \gamma^\mu\right)\gamma_\mu F_1(q^2)\right]$$

$$- C\frac{(p'-p)^\nu}{4m} F_2(q^2)\,\mathrm{Tr}\left[\left(m\gamma^\mu \not{p}' + k^\mu \not{p}\not{p}' + m\not{p}\gamma^\mu\right)\left(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu\right)\right]$$

We will consider the first term of this eq. first.

$$T_1 = C F_1(q^2)\,\mathrm{Tr}\left[p_\kappa p'_\lambda \gamma^\kappa\gamma^\mu\gamma^\lambda\gamma_\mu + m k^\mu p^\nu \gamma_\nu\gamma_\mu + m k^\mu p'^\nu \gamma_\nu\gamma_\mu + m^2\gamma^\mu\gamma_\mu\right]$$

$$= C F_1(q^2)\left\{p_\kappa p'_\lambda \mathrm{Tr}\left[(2-d)\gamma^\kappa\gamma^\lambda\right] + m k^\mu p^\nu (4g_{\mu\nu}) + m k^\mu p'^\nu (4g_{\mu\nu}) + 4dm^2\right\}$$

$$= C F_1(q^2)\left\{p_\kappa p'_\lambda (2-d)(4g^{\kappa\lambda}) + 4m(k\cdot p) + 4m(k\cdot p') + 4dm^2\right\}$$

$$= C F_1(q^2)\left\{4(2-d)(p\cdot p') + 4m(k\cdot p) + 4m(k\cdot p') + 4dm^2\right\}$$

$$(k\cdot p) = \frac{(d-2)q^2 + 4m^2}{q^2 - 4m^2}\;\frac{(p\cdot p') + m^2}{2m}$$

$$(k\cdot p') = \frac{(d-2)q^2 + 4m^2}{q^2 - 4m^2}\;\frac{(p\cdot p') + m^2}{2m}$$

$$\Bigg) =$$

$$= 4C F_1(q^2)\left\{(2-d)(p\cdot p') + 2m(k\cdot p) + dm^2\right\}$$

$$= 4C F_1(q^2)\left\{\frac{(d-2)q^2 + 4m^2}{q^2 - 4m^2}\left((p\cdot p') + m^2\right) - d(p\cdot p') + 2(p\cdot p') + dm^2\right\}$$

$$= 4C F_1(q^2)\left\{\frac{(d-2)q^2 + 4m^2}{q^2 - 4m^2}\left(2m^2 - q^2/2\right) - d\left(m^2 - q^2/2\right) + 2\left(m^2 - q^2/2\right) + dm^2\right\}$$

$$= 2C F_1(q^2)\left\{(2-d)q^2 \underline{-4m^2} \underline{-2dm^2} \underline{+q^2 d} \underline{+4m^2} - 2q^2 \underline{+2dm^2}\right\}$$

$$= 2C F_1(q^2)\left\{0\right\} = 0$$

Now we will calculate the second term:

$$T_2 = -C\frac{(p'-p)^\nu}{4m} F_2(q^2)\,\mathrm{Tr}\Big[m p'^\kappa \gamma^\mu\gamma_\kappa(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$$
$$+ k^\mu p^\kappa p'^\lambda \gamma_\kappa\gamma_\lambda(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu) + m p^\kappa \gamma_\kappa\gamma^\mu(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)\Big]$$

$$= -C\frac{(p'-p)^\nu}{4m} F_2(q^2)\Big\{ m p'^\kappa\,\mathrm{Tr}\big[(2-d)\gamma_\kappa\gamma_\nu - (d-2)\gamma_\kappa\gamma_\nu - 2\gamma_\nu\gamma_\kappa\big]$$
$$+ 4k^\mu p^\kappa p'^\lambda\big(\cancel{g_{\kappa\lambda}g_{\mu\nu}} + g_{\kappa\nu}g_{\lambda\mu} - g_{\kappa\mu}g_{\nu\lambda}$$
$$- \cancel{g_{\kappa\lambda}g_{\mu\nu}} + g_{\kappa\nu}g_{\lambda\mu} - g_{\kappa\mu}g_{\nu\lambda}\big)$$
$$+ m p^\kappa\,\mathrm{Tr}\big[d\gamma_\kappa\gamma_\nu - \gamma_\kappa(2-d)\gamma_\nu\big]\Big\}$$

$$= -C\frac{(p'-p)^\nu}{4m} F_2(q^2)\Big\{ m p'^\kappa(2-d)(4g_{\kappa\nu}) - d m p'^\kappa(4g_{\kappa\nu})$$
$$-8(k\cdot p)p'_\nu + 8(k\cdot p')p_\nu + m p^\kappa d(4g_{\kappa\nu}) - m p^\kappa(2-d)(4g_{\kappa\nu})$$

$$= -C\frac{(p'-p)^\nu}{4m} F_2(q^2)\Big\{ 8m(2-d)p'_\nu - 8mp'_\nu - 8(k\cdot p)p'_\nu$$
$$+ 8(k\cdot p')p_\nu + 4mdp_\nu + 4(d-2)mp_\nu\Big\}$$

$$= -C\frac{F_2(q^2)}{4m}\Big\{ 8m(2-d)\big(m^2-(p\cdot p')\big) - 8m\big(m^2-(p\cdot p')\big)$$
$$- 8(k\cdot p)\big(m^2-(p\cdot p')\big) + 8(k\cdot p')\big((p'\cdot p)-m^2\big)$$
$$+ 4md\big((p'\cdot p)-m^2\big) + 4m(d-2)\big((p'\cdot p)-m^2\big)\Big\}$$

$$= -C\frac{F_2(q^2)}{4m}\Big\{ 8m(2-d)\left(\frac{q^2}{2}\right) - 8m\left(\frac{q^2}{2}\right) - 8(k\cdot p)\left(\frac{q^2}{2}\right)$$
$$+ 8(k\cdot p')\left(-\frac{q^2}{2}\right) + 4md\left(-\frac{q^2}{2}\right) + 4m(d-2)\left(-\frac{q^2}{2}\right)\Big\}$$

$$= -C\frac{F_2(q^2)}{4m}\Big\{ 4m(2-d)q^2 - 4mq^2 - 4(k\cdot p)q^2 - 4(k\cdot p')q^2$$
$$-2mdq^2 - 2m(d-2)q^2\Big\}$$

$$= -C\frac{F_2(q^2)}{4m}\Big\{ 4mq^2 - 4q^2\big((k\cdot p)+(k\cdot p')\big) - 8mdq^2 + 4mq^2\Big\}$$

$$= -C\frac{F_2(q^2)}{4m}\Big\{ 8mq^2(1-d) - 8q^2\frac{(d-2)q^2+4m^2}{q^2-4m^2}\,\frac{2m^2-\frac{q^2}{2}}{2m}\Big\}$$

$$= -C \frac{F_2(q^2)}{4m} \left\{ 8mq^2(1-d) + 4q^2 \frac{(d-2)q^2 + 4m^2}{2m} \right\}$$

$$= -C \frac{F_2(q^2)}{4m} 2q^2 \left\{ 4m(1-d) + \frac{(d-2)q^2 + 4m^2}{m} \right\}$$

$$= -C \frac{F_2(q^2)}{4m} 2q^2 \left\{ \frac{4m^2 - 4m^2 d + (d-2)q^2 + 4m^2}{m} \right\}$$

$$= -C \frac{F_2(q^2)}{2m} q^2 \left\{ \frac{(d-2)q^2 + 4m^2(2-d)}{m} \right\}$$

$$= -C \frac{F_2(q^2)}{2m^2} q^2 \left\{ (d-2)(q^2 - 4m^2) \right\}$$

$$= \frac{\cancel{2m^2}}{\cancel{(d-2)}\, \cancel{q^2}\cancel{(q^2-4m^2)}} \frac{F_2(q^2)}{\cancel{2m^2}} \cancel{q^2} \cancel{(d-2)}\cancel{(q^2-4m^2)}$$

$$= F_2(q^2)$$

✓

What for, if we use the old expressio again now?
→ for higher orders (correcti e.g. up to 105 $\gamma$-matrices $(n-1)!!$ terms, where neve practical; here we could have calculated $\Gamma^\mu$ directly and identify the $(p'-p)$ term

b) $F_2(q^2) = \mathrm{Tr}\left[P_2\Gamma(p',p)\Gamma_\mu(p',p)\right]$ , where now

$$\Gamma^\mu(p',p) = -ie^2\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{\gamma^\nu(\not{p}'-\not{k}+m)\gamma^\mu(\not{p}-\not{k}+m)\gamma_\nu}{\underbrace{[(p'-k)^2-m^2+i\epsilon][(p-k)^2-m^2+i\epsilon][k^2+i\epsilon]}}$$

$\equiv D$ (denominator)

Pull trace out of integral ?

$$\rightsquigarrow F_2(q^2) = -ie^2\mu^{4-d}\frac{1}{D}\int\frac{d^dk}{(2\pi)^d}\mathrm{Tr}\Big[(\not{p}+m)\big\{C(\gamma^\mu+k^\mu)\big\}(\not{p}'+m)$$
$$\times\gamma_\nu(\not{p}'-\not{k}+m)\gamma_\mu(\not{p}-\not{k}+m)\gamma^\nu\Big]$$

$$= -ie^2\mu^{4-d}\frac{C}{D}\int\frac{d^dk}{(2\pi)}\tilde{T}\text{, where}$$

$$\tilde{T} = \mathrm{Tr}\big[(\not{p}+m)(\gamma^\mu+k^\mu)(\not{p}'+m)\gamma_\nu(\not{p}'-\not{k}+m)\gamma_\mu(\not{p}-\not{k}+m)\gamma^\nu$$

$\tilde{T}$ will be evaluated with Mathematica, yielding :

$$\tilde{T} = -\frac{8m^2}{(4m^2-q^2)^2}\Big\{(2-d)(k\cdot p)^2+(k\cdot p)\left(4m^2-2d(k\cdot p')\right)$$
$$+(k\cdot p')\left(4m^2+(2-d)(k\cdot p')\right)+4k^2m^2$$
$$-4m^2\frac{(k\cdot q)^2}{q^2}-q^2\left(k(p'+p)+k^2\right)\Big\}$$

this k here is unequal to $k^\mu$ defined before. This $k^\mu$ is already inserted !

$$= -\frac{8m^2}{(4m^2-q^2)^2}\Big\{(4m^2-q^2)k^2+(4m^2-q^2)k(p'+p)-4m^2\frac{(kq)^2}{q^2}$$
$$+(k\cdot p')\left(-2d(k\cdot p)+(2-d)(k\cdot p')\right)$$
$$+(k\cdot p)(2-d)(k\cdot p)\Big\}$$

$$= -\frac{8m^2}{(4m^2-q^2)^2}\Big\{(4m^2-q^2)\left(k^2+k(p'+p)\right)+2\left((k\cdot p')^2+(k\cdot p)^2\right)$$
$$-d\left(2(k\cdot p')(k\cdot p)+(k\cdot p')^2+(k\cdot p)^2\right)-4m^2\frac{(kq)^2}{q^2}$$

$$= -\frac{8m^2}{(4m^2-q^2)^2}\Big\{(4m^2-q^2)\left(k^2+k(p'+p)\right)+2\left((k\cdot p')^2+(k\cdot p)^2\right)$$
$$-d(k\cdot p)\left(k(p'+p)\right)-d(k\cdot p')\left(k\cdot(p'+p)\right)-4m^2\frac{(kq)^2}{q^2}\Big\}$$

$$= -\frac{8m^2}{(4m^2-q^2)^2}\Big\{(4m^2-q^2)\left(k^2+k(p'+p)\right)-d\left(k(p'+p)\right)\left(k(p'+p)\right)$$
$$+2\left((k\cdot p')^2+(k\cdot p)^2\right)-4m^2\frac{(kq)^2}{q^2}\Big\}$$

$$2(x^2+y^2) = 2\left((x+y)^2-2xy\right) = (x+y)^2 + (x+y)^2 - 4xy$$

$$= (x+y)^2 + (x-y)^2$$

$$= -\frac{8m}{(4m^2-q^2)^2}\Big\{(4m^2-q^2)\left(k^2+k(p'+p)\right) - d\left(k(p'+p)\right)^2$$

$$+\left(k(p'+p)\right)^2 + \left(k(p'-p)\right)^2 - 4m^2\frac{(kq)^2}{q^2}\Big\}$$

$$= -\frac{8m}{(4m^2-q^2)^2}\Big\{(4m^2-q^2)\left(k^2+k(p'+p)\right) - (d-1)\left(k(p'+p)\right)^2$$

$$-\frac{(kq)^2}{q^2}(4m^2-q^2)\Big\}$$

$$= \frac{8m}{(q^2-4m^2)}\Big\{k^2 + k(p'+p) + (d-1)\frac{\left(k(p'+p)\right)^2}{q^2-4m^2} - \frac{(kq)^2}{q^2}\Big\}$$

c) $$2k(p+p') = 2k^2 - [(p-k)^2-m^2] - [(p'-k)^2-m^2]$$

$$2kq = [(p-k)^2-m^2] - [(p'-k)^2-m^2]$$

$$\rightsquigarrow f_2(k) = \frac{8m^2}{q^2-4m^2}\left\{k(p+p') + k^2 + (d-1)\frac{(k(p+p'))^2}{q^2-4m^2} - \frac{(kq)^2}{q^2}\right\}$$

$$= \frac{4m^2}{q^2-4m^2}\Big\{4k^2 - [(p-k)^2-m^2] - [(p'-k)^2-m^2]$$

$$+ (d-1)\frac{k(p+p')}{q^2-4m^2}\left(2k^2 - [(p-k)^2-m^2] - [(p'-k)^2-m^2]\right)$$

$$- \frac{kq}{q^2}\left([(p-k)^2-m^2] - [(p'-k)^2-m^2]\right)\Big\}$$

d)

What is $B_0$?

$$F_2(q^2) = -ie^2\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{f_2(k)}{\underbrace{[(p'-k)^2-m^2+i\epsilon]}_{=D_1}\underbrace{[(p-k)^2-m^2+i\epsilon]}_{=D_2}\underbrace{[k^2+i\epsilon]}_{=D_3}}$$

$$= -ie^2\mu^{4-d}\left(\frac{4m^2}{q^2-4m^2}\right)\Big\{\int\frac{d^dk}{(2\pi)^d}\frac{4}{[(k-p')^2-m^2+i\epsilon][(k-p)^2-m^2+i\epsilon]}$$

$B_0(x,y,z) = B_0(x,z,y)$?

$$- \int\frac{d^dk}{(2\pi)^d}\frac{1}{[(k-p')^2-m^2+i\epsilon][k^2+i\epsilon]} - \int\frac{d^dk}{(2\pi)^d}\frac{1}{[(k-p)^2-m^2+i\epsilon][k^2+i\epsilon]}$$

$$+ \frac{(d-1)}{q^2-4m^2}(p+p')_\mu\Big\{\int\frac{d^dk}{(2\pi)^d}\frac{2k^\mu}{[(k-p')-m^2+i\epsilon][(k-p)^2-m^2+i\epsilon]}$$

$$k^\mu \mapsto k^\mu + p'^\mu$$

$$- \int\frac{d^dk}{(2\pi)^d}\frac{k^\mu}{[(k-p')^2-m^2+i\epsilon][k^2+i\epsilon]}$$

$$- \int\frac{d^dk}{(2\pi)^d}\frac{k^\mu}{[(k-p)^2-m^2+i\epsilon][k^2+i\epsilon]}\Big\}$$

$$- \frac{q_\mu}{q^2}\left\{\int\frac{d^dk}{(2\pi)^d}\frac{k^\mu}{[(k-p')^2-m^2+i\epsilon][k^2+i\epsilon]} - \int\frac{d^dk}{(2\pi)^d}\frac{k^\mu}{[(k-p)^2-m^2+i\epsilon][k^2+i\epsilon]}\right\}\Big\}$$

$$= -ie^2\mu^{4-d}\left(\frac{4m^2}{q^2-4m^2}\right)\Big\{\frac{4i}{16\pi^2}B_0(q^2,m,m) - \frac{i}{16\pi^2}B_0(m^2,0,m)$$

$$- \frac{i}{16\pi^2}B_0(m^2,0,m) + \frac{(d-1)}{q^2-4m^2}(p+p')_\mu\Big\{-\frac{2iq^\mu}{32\pi^2}B_0(q^2,m,m)$$

$$+ 2p'^\mu\frac{i}{16\pi^2}B_0(q^2,m,m) - \frac{ip'^\mu}{32\pi^2}B_0(m^2,0,m) - \frac{ip^\mu}{32\pi^2}B_0(m^2,0,m)\Big\}$$

$$- \frac{q_\mu}{q^2}\left\{\frac{ip'^\mu}{32\pi^2}B_0(m^2,0,m) - \frac{ip^\mu}{32\pi^2}B_0(m^2,0,m)\right\}\Big\}$$

$$= \frac{-ie^2\mu^{4-d}}{16\pi^2}\,\frac{4m^2}{q^2-4m^2}\Big\{4iB_0(q^2,m,m) - 2iB_0(m^2,0,m)$$

$$+ \frac{(d-1)}{q^2-4m^2}\Big\{-i\big(q(p'+p)\big)B_0(q^2,m,m) + 2i\big(m^2+(p'p)\big)B_0(q^2,m,m)$$

$$-\frac{i}{2}\big(m^2+(p'p)\big)B_0(m^2,0,m) - \frac{i}{2}\big(m^2+(p'p)\big)B_0(m^2,0,m)\Big\}$$

$$-\frac{1}{2q^2}\Big\{i(q\cdot p')B_0(m^2,0,m) - i(q\cdot p)B_0(m^2,0,m)\Big\}\Big\}$$

$$q(p'+p) = (p'-p)(p'+p) = m^2 - m^2 - pp' + pp' = 0$$

$$= \frac{-ie^2\mu^{4-d}}{16\pi^2}\,\frac{4m^2}{q^2-4m^2}\Big\{4iB_0(q^2,m,m) - 2iB_0(m^2,0,m)$$

$$+ \frac{(d-1)}{q^2-4m^2}\Big\{2i\big(2m^2-q^2/2\big)B_0(q^2,m,m) - i\big(2m^2-q^2/2\big)B_0(m^2,0,m)\Big\}$$

$$-\frac{i}{2}B_0(m^2,0,m)\Big\}$$

$$= \frac{e^2\mu^{4-d}}{16\pi^2}\,\frac{4m^2}{q^2-4m^2}\Big\{4B_0(q^2,m,m) - 2B_0(m^2,0,m)$$

$$+ \frac{(d-1)}{q^2-4m^2}\Big\{-(q^2-4m^2)B_0(q^2,m,m) + \frac{q^2-4m^2}{2}B_0(m^2,0,m)\Big\}$$

$$-\frac{1}{2}B_0(m^2,0,m)\Big\}$$

$d=4$

$$\downarrow = \frac{e^2}{16\pi^2}\,\frac{4m^2}{q^2-4m^2}\Big\{4B_0(q^2,m,m) - 2B_0(m^2,0,m) + 3\Big\{-B_0(q^2,m,m)$$

$$+\frac{1}{2}B_0(m^2,0,m)\Big\} - \frac{1}{2}B_0(m^2,0,m)\Big\}$$

$$= \frac{e^2}{16\pi^2}\,\frac{4m^2}{q^2-4m^2}\Big\{B_0(q^2,m,m) - B_0(m^2,0,m)\Big\}$$

✓

Why now $d=4$?
$\Rightarrow$ here, we
could have
set $d=4$ from
the beginning
(finite), as in
Leading order, $F_2(q$
term doesn't contain
divergences

e) $$F_2(q^2) = \frac{e^2}{16\pi^2}\frac{4m^2}{q^2-4m^2}\}\underbrace{B_0(q^2,m,m) - B_0(m^2,0,m)}_{2(y-1)\,G(y)}\{$$

Where from this arctan relation?

$$\text{with } y = \frac{4m^2}{q^2}\,,\quad G(y) = -\frac{1}{\sqrt{y-1}}\arctan\left(\frac{1}{\sqrt{y-1}}\right)$$

$$\rightsquigarrow \lim_{q\to0} \mathrel{\hat{=}} \lim_{y\to\infty}$$

$$\rightsquigarrow \lim_{q\to0} F_2(q^2) = \frac{e^2}{16\pi^2}\lim_{q\to0}\left(\frac{4m^2}{q^2-4m^2}\right)\lim_{y\to\infty}\left(2(y-1)\frac{1}{\sqrt{y-1}}\arctan\left(\frac{1}{\sqrt{y-1}}\right)\right)$$

Split limits?

$$= \frac{e^2}{8\pi^2}\underbrace{\lim_{y\to\infty}\left((y-1)\frac{1}{\sqrt{y-1}}\arctan\frac{1}{\sqrt{y-1}}\right)}$$

$$\underset{\frac{1}{\sqrt{y-1}}=x}{=}\lim_{x\to0}\frac{\arctan x}{x} \underset{\text{l'hôpital}}{=} \lim_{x\to0}\frac{1}{x^2+1} = 1$$

$$= \frac{e^2}{8\pi^2} = \frac{\alpha}{2\pi}\,,\quad \alpha = \frac{e^2}{4\pi}$$

Alternative: Taylor the expression $\arctan\frac{1}{\sqrt{y-1}}$ in $\frac{1}{\sqrt{y-1}}\approx 0$

$$\rightsquigarrow \frac{4m^2}{q^2-4m^2}\,2\,(y-1)\left(-\frac{1}{\sqrt{y-1}}\arctan\left(\frac{1}{\sqrt{y-1}}\right)\right)$$

$$= \frac{1}{\frac{1}{y}-1}\,2(1-y)\frac{1}{\sqrt{y-1}}\arctan\left(\frac{1}{\sqrt{y-1}}\right)$$

$$= \frac{2y}{\sqrt{y-1}}\left(\frac{1}{\sqrt{y-1}} + \mathcal{O}\left(\frac{1}{y^2}\right)\right) \to \frac{2y}{y-1} = 2\left(1+\frac{1}{y-1}\right) = 2$$

Why connected
ke $g = 2(1+F_2(0))$?
⇝ Peskin Schroeder, Chapter 6

f) We thus find $F_2^{\text{theo}}(0) = 0{,}001161714913$
which differs from the experimental value
$F_2^{\text{exp}}(0) = \frac{g-2}{2} = 0{,}0011596524(4)$ by about $0{,}2\%$