Disclaimer

The solution at hand was written in the course of the respective class at the University of Bonn. If not stated differently on top of the first page or the following website, the solution was prepared and handed in solely by me, Marvin Zanke. Anything in a different color than the ball pen blue is usually a correction that I or a tutor made. For more information and all my material, check:
https://www.physics-and-stuff.com/

I raise no claim to correctness and completeness of the given solutions! This equally applies to the corrections mentioned above.

11.12.2017 Advanced Quantum Field theory Exercise 9 Marvin Zaske

PS11)

a) • first way: $\frac{d}{dt}\rho(t,x) = \frac{\partial}{\partial t}\rho(t,x) + \frac{\partial x}{\partial t}\frac{\partial}{\partial x}\rho(t,x)$

$= \frac{\partial}{\partial t}\rho(t,x) + v(x)\frac{\partial}{\partial x}\rho(t,x)$

and $\frac{d}{dt}\rho(t,x) = g(x)\rho(t,x)$

$\rightsquigarrow \left\{\frac{\partial}{\partial t} + v(x)\frac{\partial}{\partial x} - g(x)\right\}\rho(t,x) = 0$

g(x) growth rate per bacteria density? → like this in tutorial ✓

• alternative: $\frac{\partial}{\partial t}\rho(t,x) = g(x)\rho(t,x) - v(x)\frac{\partial}{\partial x}\rho(t,x)$

↑ change w/ time; ↑ growth rate (new bacteria); ↑ bacteria flowing away from x and flowing in to x

b) $\frac{d}{dt}\bar{x}(t,x) = v(\bar{x})$ w/ $\bar{x}(0,x) = x$

Where from this diff. eq.?

Want to show that $\rho(t,x) = \rho_0(\bar{x}(-t,x))\exp\left\{\int_0^t dt' g(\bar{x}(-t',x))\right\}$ ✓ Should be different t's? → Bullshit

Solves this diff-eq

$0 = \left\{\frac{\partial}{\partial t} + v(x)\frac{\partial}{\partial x} - g(x)\right\}\rho(t,\bar{x}) = \left\{\frac{\partial}{\partial t} + \frac{d\bar{x}(t,x)}{dt}\frac{\partial}{\partial \bar{x}} - g(x)\right\}\rho(t,\bar{x})$

$= \left\{\frac{d}{dt} - g(\bar{x})\right\}\rho(t,\bar{x}) \Leftrightarrow \frac{d}{dt}\rho(t,\bar{x}) = g(\bar{x})\rho(t,\bar{x})$

different in lecture?

$\Leftrightarrow \frac{\frac{d}{dt}\rho(t,\bar{x})}{\rho(t,\bar{x})} = g(\bar{x}) \Leftrightarrow \frac{d}{dt}\log\rho(t,\bar{x}) = g(\bar{x})$

$\rightsquigarrow \int_0^t dt' \frac{d}{dt}\log\rho(t',\bar{x}) = \int_0^t dt' g(\bar{x})$

Only working if those new variables exactly given like this?

$\Leftrightarrow \log(\rho(t,\bar{x})) - \log(\rho(0,x)) = \int_0^t dt' g(\bar{x})$

$\Leftrightarrow \rho(t,\bar{x}(t,x)) = \rho(0,\bar{x}(0,x))\exp\left\{\int_0^t dt' g(\bar{x}(0,x))\right\}$

still $\bar{x}(t,x) \rightarrow \bar{x}(0,x)$

$\rho(t,x) = \rho_0(\bar{x}(-t,x))\exp\left\{\int_0^t dt' g(\bar{x}(-t,x))\right\}$