Theoretical Particle Physics 2 Homework 1 — Marvin Zanke

15.04.2018

1)

a) $$\mathcal{L}_{gq} = -\frac{g}{\sqrt{2}}\left\{\overline{u_{iL}}\gamma^\mu W^+_\mu d_{iL} + \overline{d_{iL}}\gamma^\mu W^-_\mu u_{iL}\right\}$$

$$-\frac{g}{\cos\theta_W} Z_\mu \left\{\overline{u_{iL}}\gamma^\mu u_{iL} C_L^{(u)} + \overline{d_{iL}}\gamma^\mu d_{iL} C_L^{(d)} + \overline{u_{iR}}\gamma^\mu u_{iR} C_R^{(u)} + \overline{d_{iR}}\gamma^\mu d_{iR} C_R^{(d)}\right\}$$

where from the $\mathcal{L}_{gq}$-term?
→ from $\sum_j \overline{(q_L)_j}\, i\not{D}(q_L)_j + \sum_f \overline{l_f}\, i\not{D}\, l_f + \sum_j \overline{(q_R)_j}\, i\not{D}(q_R)_j$
$\mathcal{D}$ → ? part

Coupling between right/left-handed in mass-term?
→ $m\bar{\psi}\psi$ with $\psi = \psi_L + \psi_R$

$$\mathcal{L}_{q\text{-mass}} = -\sum_{q=u,d} \overline{q_{jR}}\,(M_q)_{jk}\, q_{kL} + h.c.$$

where the indices denote the flavor generation (u,d like) and left-/right-handed respectively.

Using U^L_q and U^R_q (unitarity trafos) in order to diagonalize M_q and (re)defining
$$q'_L = U^L_q q_L\,, \quad q'_R = U^R_q q_R$$
as well as the fact that $U^{L,R}_q$ and the γ-matrices act in different spaces and thus commute, we find

q_R etc all spinors themselves?
→ yes, are 4-comp spinors, while U acts in flavor space $SU(2)_F \cap SU(2)_{sp} = \emptyset$

M comp. or also vector form possible?
→ in tutorial e.g. $(\overline{q_R}(U^R)^\dagger)_j$

$$\mathcal{L}_{q\text{-mass}} \overset{U^\dagger_{ij}\text{ denotes } i,j \text{ comp. of } U^\dagger}{=} -\sum_{q=u,d} \overline{q_{jR}}\,(U^R_q)^\dagger_{j\ell}(U^R_q)_{\ell a}(M_q)_{ab}(U^L_q)^\dagger_{bm}(U^L_q)_{mk}\, q_{kL} + h.c.$$

$\overline{U_{ij}q_j} = \overline{q_j}\,U^\dagger_{ji} \rightarrow$
$$= -\sum_{q=u,d} \overline{(U^R_q)_{\ell j} q_{jR}}\,(U^R_q)_{\ell a}(M_q)_{ab}(U^L_q)^\dagger_{bm}(U^L_q)_{mk}\, q_{kL}$$

$$= -\sum_{q=u,d} \overline{q'_{\ell R}}\,\underbrace{(U^R_q)_{\ell a}(M_q)_{ab}(U^L_q)_{bm}}_{(M^{diag}_q)_{\ell m}}\, q'_{mL}$$

Why can it always be diag. like this (2 different matrices)?
→ otherwise no trafo to get to phys. mass eigenstates like this because general for a matrix that 2 different matrices.

Inserting those relations in $\mathcal{L}_{gq}$, we find

$$\mathcal{L}_{gq} = -\frac{g}{\sqrt{2}}\left\{\overline{u'_{kL}}(U^L_u)_{ki}\gamma^\mu W^+_\mu (U^L_d)^\dagger_{im} d'_{mL} + \overline{d'_{kL}}(U^L_d)_{ki}\gamma^\mu W^-_\mu (U^L_u)^\dagger_{im} u'_{mL}\right\}$$

$$-\frac{g}{\cos\theta_W} Z_\mu \left\{\overline{u'_{kL}}(U^L_u)_{ki}\gamma^\mu (U^L_u)^\dagger_{im} u'_{mL} C_L^{(u)} + \overline{d'_{kL}}(U^L_d)_{ki}\gamma^\mu (U^L_d)^\dagger_{im} d'_{mL} C_L^{(d)} + \overline{u'_{kR}}(U^R_u)_{ki}\gamma^\mu (U^R_u)^\dagger_{im} u'_{mR} C_R^{(u)} + \overline{d'_{kR}}(U^R_d)_{ki}\gamma^\mu (U^R_d)^\dagger_{im} d'_{mR} C_R^{(d)}\right\}$$

using that $(U^L_q)_{ki}(U^L_q)^\dagger_{im} = \delta_{km} = (U^R_q)_{ki}(U^R_q)^\dagger_{im}$, we find

$$\mathcal{L}_{qq} = -\frac{g}{\sqrt{2}}\left\{\overline{u'_{kL}}\gamma^\mu W^+_\mu \, U_{km}\, d'_{mL} + \overline{d'_{kL}}\gamma^\mu W^-_\mu \, U^\dagger_{km}\, u'_{mL}\right\}$$
$$-\frac{g}{\cos\theta_W} Z_\mu \left\{\overline{u'_{kL}}\gamma^\mu u'_{kL}\, c_L^{(u)} + \overline{d'_{kL}}\gamma^\mu d'_{kL}\, c_L^{(d)} + \overline{u'_{kR}}\gamma^\mu u'_{kR}\, c_R^{(u)} + \overline{d'_{kR}}\gamma^\mu d'_{kR}\, c_R^{(d)}\right\}$$

where $U \equiv (U^L_u)(U^L_d)^\dagger \rightsquigarrow U^\dagger = (U^L_d)(U^L_u)^\dagger$ KM-matrix

✓ KM-matrix between spinors?
⇝ yes, between vectors w/ indices

✓ why only U for left-handed and no right-handed?
⇝ no flavor change possible for right-handed particles

b) $$\mathcal{L}_{l.i} = -\overline{\nu_{iL}}\left(\frac{g}{2}W_{3\mu} + g' Y_L B_\mu\right)\gamma^\mu \nu_{iL} - \overline{e_{iL}}\,\frac{g}{2}\underbrace{(W_{1\mu} + iW_{2\mu})}_{\sqrt{2}W_\mu^-}\gamma^\mu \nu_{iL}$$
$$-\overline{\nu_{iL}}\,\frac{g}{2}\underbrace{(W_{1\mu} - iW_{2\mu})}_{\sqrt{2}W_\mu^+}\gamma^\mu e_{iL} - \overline{e_{iL}}\left(g' Y_L B_\mu - \frac{g}{2}W_{3\mu}\right)\gamma^\mu e_{iL}$$

e_i, ν_i denote the different flavors

$$+ g'\,\overline{e_{iR}}\, B_\mu \gamma^\mu e_{iR}$$

$$= -\overline{\nu_{iL}}\left\{\frac{g}{2}(\cos\theta_W Z_\mu + \sin\theta_W A_\mu) + g' Y_L(\cos\theta_W A_\mu - \sin\theta_W Z_\mu)\right\}\gamma^\mu \nu_{iL}$$
$$-\overline{e_{iL}}\,\frac{g}{\sqrt{2}}W_\mu^-\gamma^\mu \nu_{iL} - \nu_{iL}\,\frac{g}{\sqrt{2}}W_\mu^+\gamma^\mu e_{iL}$$
$$-\overline{e_{iL}}\left\{g' Y_L(\cos\theta_W A_\mu - \sin\theta_W Z_\mu) - \frac{g}{2}(\cos\theta_W Z_\mu + \sin\theta_W A_\mu)\right\}\gamma^\mu e_{iL}$$
$$+ g'\,\overline{e_{iR}}(\cos\theta_W A_\mu - \sin\theta_W Z_\mu)\gamma^\mu e_{iR}$$

✓ "M_l and hence the matrix of lepton Yuk. cpl. is diagonal" Why use those diag. hoc - transitions between different families?
⇝ phys. (mass) basis and flavor basis diagonal → no transitions between fam. [illegible]
Matrices remain in mass term?

$Y_L = -1/2$

$e = g'\cos\theta_W = g\sin\theta_W$

$$= -\overline{\nu_{iL}}\left\{A_\mu(e/2 - e/2) + Z_\mu\left(\frac{g}{2}\cos\theta_W + \frac{g'}{2}\sin\theta_W\right)\right\}\gamma^\mu \nu_{iL}$$
$$-\overline{e_{iL}}\,\frac{g}{\sqrt{2}}W_\mu^-\gamma^\mu \nu_{iL} - \nu_{iL}\,\frac{g}{\sqrt{2}}W_\mu^+\gamma^\mu e_{iL}$$
$$-\overline{e_{iL}}\left\{A_\mu(-e/2 - e/2) - Z_\mu\left(-g'/2\,\sin\theta_W + g/2\,\cos\theta_W\right)\right\}\gamma^\mu e_{iL}$$
$$+ g'\,\overline{e_{R}}(\cos\theta_W A_\mu - \sin\theta_W Z_\mu)\gamma^\mu e_{iR}$$

$$= -\overline{\nu_{iL}} Z_\mu\left(\frac{g}{2}\cos\theta_W + \frac{g'}{2}\sin\theta_W\right)\gamma^\mu \nu_{iL}$$
$$-\overline{e_{iL}}\,\frac{g}{\sqrt{2}}W_\mu^-\gamma^\mu \nu_{iL} - \overline{\nu_{iL}}\,\frac{g}{\sqrt{2}}W_\mu^+\gamma^\mu e_{iL}$$
$$-\overline{e_{iL}}\left\{-eA_\mu + Z_\mu\left(\frac{g'}{2}\sin\theta_W - \frac{g}{2}\cos\theta_W\right)\right\}\gamma^\mu e_{iL}$$
$$+\overline{e_{iR}}(eA_\mu - g'\sin\theta_W Z_\mu)\gamma^\mu e_{iR}$$

Where are right-handed positrons? And no left-handed electron with positron vertex?

✓ shorter form will [illegible] [illegible]
⇝ just use proportionalities e.g. $W_\mu^\pm$ [illegible]

Looking at

$$\mathcal{L}_{e\text{-mass}} = -\overline{l^-_{jR}}\,(\mathcal{M}_e)_{jk}\,l_{kL} + h.c.$$

No (-) on 2nd lepton l^-_{kL}?

and using V^L and V^R to diagonalize $\mathcal{M}_e$ s.t.

$$l^-_R = V^R l^-_R \quad \text{and} \quad l^-_L = V^L l^-_L$$

we find

$$\begin{aligned}
\mathcal{L}_{e\text{-mass}} &= -\overline{l^-_{jR}}\,(V^R)^\dagger_{jl}\,(V^R)_{la}\,(\mathcal{M}_e)_{ab}\,(V^L)^\dagger_{bm}\,(V^L)_{mk}\,l^-_{kL} \\
\overline{V_{ij} l_j} = \bar{l}_j V^\dagger_{ji} \searrow \; &= -\overline{(V^R)_{lj}\,e^-_{jR}}\,(V^R)_{la}\,(\mathcal{M}_e)_{ab}\,(V^L)^\dagger_{bm}\,(V^L)_{mk}\,l^-_{kL} \\
&= -\overline{e^-_{lR}}\,(V^R)_{la}\,(\mathcal{M}_e)_{ab}\,(V^L)^\dagger_{bm}\,l^-_{m_L}
\end{aligned}$$

We thus find

$$\begin{aligned}
\mathcal{L}_{e.i.} = &-\overline{\nu_{iL}}\,Z_\mu\left(\frac{g}{2}\cos\theta_W + \frac{g'}{2}\sin\theta_W\right)\gamma^\mu\,\nu_{iL} \\
&- \overline{(V^L)^\dagger_{ik}\,e'^-_{kL}}\,\frac{g}{\sqrt{2}}\,W^-_\mu\,\gamma^\mu\,\nu_{iL} - \overline{\nu_{iL}}\,\frac{g}{\sqrt{2}}\,W^+_\mu\,\gamma^\mu\,(V^L)^\dagger_{ik}\,e'^-_{kL} \\
&- \overline{(V^L)^\dagger_{ik}\,e'^-_{kL}}\left\{-eA_\mu + Z_\mu\left(\frac{g'}{2}\sin\theta_W - \frac{g}{2}\cos\theta_W\right)\right\}\gamma^\mu\,(V^L)_{im}\,e'^-_{mL} \\
&+ \overline{(V^R)^\dagger_{ik}\,e'^-_{kR}}\,(eA_\mu - g'\sin\theta_W Z_\mu)\,\gamma^\mu\,(V^R)_{im}\,e'^-_{mR} \\
= &-\overline{\nu_{iL}}\,Z_\mu\left(\frac{g}{2}\cos\theta_W + \frac{g'}{2}\sin\theta_W\right)\gamma^\mu\,\nu_{iL} \\
&- \overline{e'^-_{kL}}\,(V^L)_{ki}\,\frac{g}{\sqrt{2}}\,W^-_\mu\,\gamma^\mu\,\nu_{iL} - \overline{\nu_{iL}}\,\frac{g}{\sqrt{2}}\,W^+_\mu\,\gamma^\mu\,(V^L)^\dagger_{ik}\,e'^-_{kL} \\
&- \overline{e'^-_{kL}}\,(V^L)_{ki}\left\{-eA_\mu + Z_\mu\left(\frac{g'}{2}\sin\theta_W - \frac{g}{2}\cos\theta_W\right)\right\}\gamma^\mu\,(V^L)_{im}\,e'^-_{mL} \\
&+ \overline{e'^-_{kR}}\,(V^R)_{ki}\,(eA_\mu - g'\sin\theta_W Z_\mu)\,\gamma^\mu\,(V^R)_{im}\,e'^-_{mR}
\end{aligned}$$

No definition for ν_R trafo?

using $(V^L)_{ki}\,(V^L)_{im} = \delta_{km}$ and applying

$$\nu'_L = V^L \nu_L \Leftrightarrow \nu_L = (V^L)^\dagger \nu'_L$$ to the neutrinos, we find

$$\begin{aligned}
\mathcal{L}_{e.i.} = &-\overline{\nu'_{kL}}\,(V^L)_{ki}\,Z_\mu\left(\frac{g}{2}\cos\theta_W + \frac{g'}{2}\sin\theta_W\right)\gamma^\mu\,(V^L)^\dagger_{im}\,\nu'_{mL} \\
&- \overline{e'^-_{kL}}\,(V^L)_{ki}\,\frac{g}{\sqrt{2}}\,W^-_\mu\,\gamma^\mu\,(V^L)^\dagger_{im}\,\nu'_{mL} \\
&- \overline{\nu'_{mL}}\,(V^L)_{mi}\,\frac{g}{\sqrt{2}}\,W^+_\mu\,\gamma^\mu\,(V^L)^\dagger_{ik}\,e'^-_{kL} \\
&- \overline{e'^-_{kL}}\left\{-eA_\mu + Z_\mu\left(\frac{g'}{2}\sin\theta_W - \frac{g}{2}\cos\theta_W\right)\right\}\gamma^\mu\,e'_{kL} \\
&+ \overline{e'^-_{kR}}\,(eA_\mu - g'\sin\theta_W\,Z_\mu)\,\gamma^\mu\,e'^-_{kR}
\end{aligned}$$

$$
\begin{aligned}
= &- \overline{\nu'_{kL}}\, Z_\mu \left(\frac{g}{2} \cos\theta_W + \frac{g'}{2} \sin\theta_W \right) \gamma^\mu \nu'_{kL} \\
&- \overline{e^{-\prime}_{kL}}\, \frac{g}{\sqrt{2}} W^-_\mu \gamma^\mu \nu'_{kL} - \overline{\nu'_{kL}}\, \frac{g}{\sqrt{2}} W^+_\mu \gamma^\mu e^{-\prime}_{kL} \\
&- \overline{e^{-\prime}_{kL}} \left\{ -eA_\mu + Z_\mu \left(\frac{g'}{2} \sin\theta_W - \frac{g}{2} \cos\theta_W \right) \right\} \gamma^\mu e^{-\prime}_{kL} \\
&+ \overline{e^{-\prime}_{kR}} \left(eA_\mu - g' \sin\theta_W Z_\mu \right) \gamma^\mu e^{-\prime}_{kR}
\end{aligned}
$$

2) $\Psi = \begin{pmatrix} \xi_L \\ \xi_R \end{pmatrix}$, $P_L = \frac{1-\gamma_5}{2}$, $P_R = \frac{1+\gamma_5}{2}$

a) $\gamma_5 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}$ in chiral basis, $\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$

$\rightsquigarrow P_L \Psi = \begin{pmatrix} \xi_L \\ 0 \end{pmatrix}$, $P_R \Psi = \begin{pmatrix} 0 \\ \xi_R \end{pmatrix}$.

Quark and lepton mass term cannot be of this form (see lecture)?

$$\mathcal{L}_{\text{Diracmass}} = -m \overline{\Psi}_R \Psi_L + \text{h.c.}$$
$$= -m\left(\Psi_R^\dagger \gamma^0 \Psi_L + \Psi_L^\dagger \gamma^0 \Psi_R\right)$$
$$= -m\left(\begin{pmatrix} 0 \\ \xi_R \end{pmatrix}^\dagger \gamma^0 \begin{pmatrix} \xi_L \\ 0 \end{pmatrix} + \begin{pmatrix} \xi_L \\ 0 \end{pmatrix}^\dagger \gamma^0 \begin{pmatrix} 0 \\ \xi_R \end{pmatrix}\right)$$
$$= -m\left(\begin{pmatrix} 0 \\ \xi_R \end{pmatrix}^\dagger \begin{pmatrix} 0 \\ \xi_L \end{pmatrix} + \begin{pmatrix} \xi_L \\ 0 \end{pmatrix}^\dagger \begin{pmatrix} \xi_R \\ 0 \end{pmatrix}\right) = -m\left(\xi_R^\dagger \xi_L + \xi_L^\dagger \xi_R\right)$$
$$= -m\left(\xi_R^\dagger \xi_L\right) + \text{h.c.}$$

b) $\Psi^c \equiv C\overline{\Psi}^T \overset{!}{=} \Psi \xleftarrow{} $ Majorana spinor

with $C = i\gamma^2\gamma^0$, $\gamma^2 = \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}$, $\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ in chiral basis

$\Psi = \begin{pmatrix} \xi_L \\ \xi_R \end{pmatrix} \rightsquigarrow \Psi^c = i\gamma^2\gamma^0\left(\Psi^\dagger\gamma^0\right)^T$

The physical meaning of this condition is that the charge conjugation has no effect on the particle, i.e. particle and antiparticle are exactly the same (same phys. properties) and especially are neutral.

All neutral particles would be Majorana Spinors then?

$$\rightsquigarrow \Psi^c = i\gamma^2\gamma^0\left(\Psi^\dagger\gamma^0\right)^T = i\gamma^2\gamma^0\left(\gamma^0\right)^T \Psi^* = i\gamma^2\Psi^*$$
$$= i\begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}\begin{pmatrix} \xi_L \\ \xi_R \end{pmatrix}^* = i\begin{pmatrix} \sigma^2\xi_R^* \\ -\sigma^2\xi_L^* \end{pmatrix} \overset{!}{=} \begin{pmatrix} \xi_L \\ \xi_R \end{pmatrix}$$

$$\rightsquigarrow \xi_L \overset{!}{=} i\sigma^2\xi_R^*, \quad \xi_R \overset{!}{=} -i\sigma^2\xi_L^*$$

$$\rightsquigarrow \xi_L \overset{!}{=} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\xi_R^*, \quad \xi_R \overset{!}{=} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\xi_L^*$$

with $\xi_L = \begin{pmatrix} x \\ y \end{pmatrix}$, $\xi_R = \begin{pmatrix} w \\ z \end{pmatrix}$, we also get

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} w^* \\ z^* \end{pmatrix} \rightsquigarrow x = z^*, \quad y = -w^*$$

$$\xi_L = \begin{pmatrix} x \\ y \end{pmatrix}, \quad \xi_R = \begin{pmatrix} -y^* \\ x^* \end{pmatrix}$$

c) $$CC^\dagger = (i\gamma^2\gamma^0)(i\gamma^2\gamma^0)^\dagger = i\gamma^2\gamma^0(\gamma^0)^\dagger(\gamma^2)^\dagger(-i) = \gamma^2\gamma^0\gamma^0\gamma^0\gamma^2\gamma^0 = \gamma^2\gamma^0\gamma^2\gamma^0$$
$$= -\gamma^0\gamma^2\gamma^2\gamma^0 = (\gamma^0)^2 = \mathbb{1}$$

$\rightsquigarrow C^{-1} = C^\dagger$

$$C^{-1}\gamma_5 C = C^\dagger \gamma_5 C = (i\gamma^2\gamma^0)^\dagger \gamma_5 (i\gamma^2\gamma^0) = (\gamma^0)^\dagger (\gamma^2)^\dagger \gamma_5 \gamma^2 \gamma^0$$

$$= \gamma^0\gamma^0\gamma^2\gamma^0\gamma_5\gamma^2\gamma^0 = \gamma^2\gamma^0\gamma^2\gamma^0\gamma_5 = -(\gamma^2)^2\gamma_5 = \gamma_5$$

$= (\gamma_5)^T$ in chiral basis

Only chiral and Dirac basis?

$$P_R(\psi_{fL})^c = \frac{1+\gamma_5}{2} C \overline{\psi_{fL}}^T = C C^{-1} \frac{1+\gamma_5}{2} C \overline{\psi_{fL}}^T$$

$$= C \frac{1+\gamma_5}{2} \overline{\psi_{fL}}^T = C\left[\overline{\psi_{fL}} \frac{1+\gamma_5}{2}\right]^T = C[\overline{\psi_{fL}} P_R]^T$$

$$= C[\overline{P_L\psi_{fL}}]^T = C[\overline{\psi_{fL}}]^T = \psi_{fL}^c$$

$$P_L(\psi_{fL})^c = \frac{1-\gamma_5}{2} C \overline{\psi_{fL}}^T = C \frac{1-\gamma_5}{2} \overline{\psi_{fL}}^T = C[\overline{\psi_{fL}} P_L]^T$$

$$= C\left[\psi_{fL}^\dagger \gamma^0 \frac{1-\gamma_5}{2}\right]^T = C\left[\psi_{fL}^\dagger \frac{1+\gamma_5}{2}\gamma^0\right]^T$$

$$= C[\underbrace{\overline{P_R\psi_{fL}}}_{P_R P_L \psi_f}]^T = 0$$

d) $\mathcal{L}_{\text{Maj. mass}} = -\frac{1}{2} M_L (\psi_{fL})^c \psi_{fL} + h.c.$

$(\psi_L)^c$ right handed now, so not the same particle?

Noticing that with $\psi_f = \psi_{fL} + \psi_{fR}$ and $\psi_{fR} = C\overline{\psi_{fL}}^T = \psi_{fL}^c$

we find for ψ_f w/ $\overline{\psi_f}(i\not\partial - m)\psi_f$:

$$\overline{\psi_f}\psi_f = \overline{\psi_f}(P_L+P_R)(P_L+P_R) = \overline{\psi_f}(P_L^2+P_R^2)\psi_f = \overline{\psi_f}\left\{\left(\frac{1-\gamma_5}{2}\right)^2 + \left(\frac{1+\gamma_5}{2}\right)^2\right\}\psi_f$$

$$= \psi_f^\dagger \frac{1+\gamma_5}{2}\gamma^0\frac{1-\gamma_5}{2}\psi_f + \psi_f^\dagger\frac{1-\gamma_5}{2}\gamma^0\frac{1+\gamma_5}{2}\psi_f$$

$$= (P_R\psi_f)^\dagger\gamma^0 P_L\psi_f + (P_L\psi_f)^\dagger\gamma^0 P_R\psi_f = \underline{\overline{\psi_{fR}}\psi_{fL}} + \overline{\psi_{fL}}\psi_{fR}$$

$$= C\overline{\psi_{fL}}^T\psi_{fL} + h.c.$$

$$= \psi_{fL}^T C \psi_{fL} + h.c.$$

Why $\overline{\psi_{fL}}\psi_{fR}$ (doublet, si) not gauge inv

only in same rep.?

$$\overline{\psi_f}\not\partial\psi_f = \overline{\psi_f}\partial_\mu\gamma^\mu\psi_f = \overline{\psi_f}(P_L^2+P_R^2)\partial_\mu\gamma^\mu\psi_f$$

$$= \overline{\psi_f}\frac{1-\gamma_5}{2}\frac{1-\gamma_5}{2}\partial_\mu\gamma^\mu\psi_f + \overline{\psi_f}\frac{1+\gamma_5}{2}\frac{1+\gamma_5}{2}\partial_\mu\gamma^\mu\psi_f$$

$\{\gamma_5,\gamma_\mu\}=0$

$$\overset{\downarrow}{=} \overline{P_R\psi_f}\,\partial_\mu\gamma^\mu P_R\psi_f + \overline{P_L\psi_f}\,\partial_\mu\gamma^\mu P_L\psi_f = \overline{\psi_{fR}}\not\partial\psi_{fR} + \overline{\psi_{fL}}\not\partial\psi_{fL}$$

$$\overline{\psi_{fR}}\not\partial\psi_{fR} = (\overline{C\overline{\psi_{fL}}^T})\partial_\mu\gamma^\mu C\overline{\psi_{fL}}^T = \overline{\psi_{fL}}^{T\dagger} C^\dagger\gamma^0\partial_\mu\gamma^\mu C\overline{\psi_{fL}}^T$$

$$= (\psi_{fL}^\dagger\gamma^0)^* C^\dagger\gamma^0\partial_\mu\gamma^\mu C(\psi_{fL}^\dagger\gamma^0)^T$$

$$= \psi_{fL}^T(\gamma^0)^* C^\dagger\gamma^0\partial_\mu\gamma^\mu C(\gamma^0)^T\psi_{fL}^*$$

What's ∂_μ^T?

it's a scalar → can be transposed (-) from fermions

$$= -\psi_{fL}^\dagger\gamma^0 C^T\partial_\mu(\gamma^\mu)^T(\gamma^0)^\dagger C^*(\gamma^0)^\dagger\psi_{fL}$$

$$= -\overline{\psi_{fL}}(i(\gamma^0)^T(\gamma^2)^T)\partial_\mu(\gamma^\mu)^T(\gamma^0)^T(-i(\gamma^2)^*(\gamma^0)^*)\gamma^0\psi_{fL}$$

(-) sign upon transposing?

Chiral rep.

$$\overset{\downarrow}{=} -\overline{\psi_{fL}}(\gamma^0(-\gamma^2))\partial_\mu(\gamma^\mu)^T\gamma^0(-\gamma^2)\psi_{fL}$$

$$(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0,\ (\gamma^\mu)^* = \gamma^2\gamma^\mu\gamma^2 \leadsto (\gamma^\mu)^T = (\gamma^0)^*(\gamma^\mu)^*(\gamma^0)^* = \gamma^0\gamma^2\gamma^\mu\gamma^2\gamma^0$$

$$= -\overline{\psi_{fL}}\gamma^0\gamma^2\partial_\mu\gamma^0\gamma^2\gamma^\mu\gamma^2\gamma^0\gamma^0\gamma^2\psi_{fL}$$

$= 2\bar{\psi}_L \partial_\mu \gamma^\mu \psi_L$

Expressing $\bar{\psi}(i\partial\!\!\!/ - m)\psi$ in terms of ψ_L thus yields

$$\mathcal{L} = 2\psi_L \partial_\mu \gamma^\mu \psi_L - m(\psi_L^c \psi_L + h.c.)$$

$$\Leftrightarrow \mathcal{L}' = \psi_L \partial_\mu \gamma^\mu \psi_L - \frac{1}{2} m(\psi_L^c \psi_L + h.c.)$$

Shouldn't $(\psi)^c$ be $= \psi$ for $\psi = \psi_L + \psi_R$, $\psi_R = \psi_L^c$?

e) $\mathcal{L}_{maj.mass,R} = -\frac{1}{2} M_R \overline{(\psi_R)^c} \psi_R + h.c.$

We find $\psi_L^c = (P_L\psi)^c = C\overline{P_L\psi}^T = C((P_L\psi)^\dagger\gamma^0)^T$

$= C(\psi^\dagger P_L \gamma^0)^T = C(\psi^\dagger \gamma^0 P_R)^T$

$= C(\bar{\psi} P_R)^T = C P_R \bar{\psi}^T = P_R C \bar{\psi}^T$

$= P_R \psi^c = P_R \psi = \psi_R$

$\rightsquigarrow \psi_R^c = \psi_L$ as $(\psi_L^c)^c = \psi_L$

So $\psi_L^c = \psi_R = \psi_L$ as ψ_L majorana spinor or only $\psi = \psi_L + \psi_R$ majorana spinor?

Thus $\mathcal{L}_{maj.mass,R} = -\frac{1}{2} M_R \bar{\psi}_L \psi_R + h.c.$

$\mathcal{L}_{maj.mass,L} = -\frac{1}{2} M_L \bar{\psi}_R \psi_L + h.c.$

and $(\bar{\psi}_L \psi_R)^\dagger = \bar{\psi}_R \psi_L$

$\rightsquigarrow M_L = M_R$

Can also express in Components $\begin{pmatrix}\xi_L \\ \xi_R\end{pmatrix}$ and add the 2 mass terms

$\mathcal{L} = M_L \xi_L^\dagger \xi_R + M_R \xi_R^\dagger \xi_L$ (hermitian!)

$= M_L \xi_R \xi_L + M_R \xi_L^\dagger \xi_R$

why transform ψ_L separately?

f) If $\psi_L \to e^{i\alpha}\psi_L$ for $U(1)_Y$ $\Rightarrow \bar{\psi}_L = e^{-i\alpha}\bar{\psi}_L$

$\rightsquigarrow (\psi_L)^c = C\bar{\psi}^T \to e^{-i\alpha}(\psi_L)^c$

$\rightsquigarrow \overline{(\psi_L)^c}\psi_L \to e^{2i\alpha}\overline{(\psi_L)^c}\psi_L$ only invariant for $Y=0$

but no particle like this in SM. $Y(\nu_L) = -1/2 \rightsquigarrow$ Need $2\times\phi$ (one only cancels $1\times Y$)

why should it be non-renorm.? neg. cpl.?

With Higgs $Y_\phi = 1/2$ and neutrinos $Y_\nu = -1/2$, we could construct $\mathcal{L}_\nu = \overline{\nu_L^c}\,\phi\phi\,\nu_L$

$\bar{L} H \tilde{H} L^* = \frac{yv^2}{2} \psi_L \psi_L^c$

Why v.e.v. and not higgs?

$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$f_\nu \bar{L} H \tilde{H} L$ $\quad \tilde{H} = i\sigma^2 H^*$ (Before: $\bar{L} H e_R$ Yukawa)

$\rightsquigarrow f v^2 \overline{\nu_L^c}\nu_L$ $\quad H = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + h(x)) \end{pmatrix}$

or $f v^2 \overline{\psi_L^c}\psi_L$

s $Y_\phi = 1/2$ or $Y_h = 1/2$?