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Advanced Theoretical Particle Physics Homework 9 Marvin Zanke

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1) Chiral (left) Superfield $\Phi(x,\theta,\bar\theta)$ in terms of $\phi(x)$, $\xi(x)$, $F(x)$

Left-chiral?

$$\Phi(x,\theta,\bar\theta) = \phi(x) - i\theta\sigma^\mu\bar\theta\,\partial_\mu\phi(x) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\,\partial^\mu\partial_\mu\phi(x)$$
$$+\sqrt{2}\,\theta\xi(x) + \frac{i}{\sqrt{2}}\theta\theta\,\partial_\mu\xi(x)\sigma^\mu\bar\theta + \theta\theta F(x)$$

$()^\dagger$ no effect on $\theta\sigma^\mu\bar\theta$! $\Rightarrow$ See tutorial!

$$\Phi^\dagger(x,\theta,\bar\theta) = \phi^*(x) + i\theta\sigma^\mu\bar\theta\,\partial_\mu\phi^*(x) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\,\partial^\mu\partial_\mu\phi^*(x)$$
$$+\sqrt{2}\,\bar\theta\bar\xi(x) - \frac{i}{\sqrt{2}}\bar\theta\bar\theta\,\theta\sigma^\mu\partial_\mu\bar\xi(x) + \bar\theta\bar\theta F^*(x)$$

$$\Rightarrow \Phi_i^\dagger(x)\Phi_j(x) = \Big\{\phi_i^*(x) + i\theta\sigma^\mu\bar\theta\,\partial_\mu\phi_i^*(x) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\,\partial^\mu\partial_\mu\phi_i^*(x)$$
$$+\sqrt{2}\,\bar\theta\bar\xi_i(x) - \frac{i}{\sqrt{2}}\bar\theta\bar\theta\,\theta\sigma^\mu\partial_\mu\bar\xi_i(x) + \bar\theta\bar\theta F_i^*(x)\Big\}$$
$$\times\Big\{\phi_j(x) - i\theta\sigma^\nu\bar\theta\,\partial_\nu\phi_j(x) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\,\partial^\nu\partial_\nu\phi_j(x)$$
$$+\sqrt{2}\,\theta\xi_j(x) + \frac{i}{\sqrt{2}}\theta\theta\,\partial_\nu\xi_j(x)\sigma^\nu\bar\theta + \theta\theta F_j(x)\Big\}$$

terms $\sim\theta^3\bar\theta^3$ vanish

$$= \phi_i^*(x)\phi_j(x) - i\phi_i^*(x)\{\theta\sigma^\mu\bar\theta\,\partial_\mu\phi_j(x)\} - \frac{1}{4}\phi_i^*(x)\{\theta\theta\bar\theta\bar\theta\,\partial^\mu\partial_\mu\phi_j(x)\}$$
$$+\sqrt{2}\,\phi_i^*\theta\xi_j(x) + \frac{i}{\sqrt{2}}\phi_i^*(x)\{\theta\theta\,\partial_\mu\xi_j(x)\sigma^\mu\bar\theta\} + \phi_i^*(x)\theta\theta F_j(x)$$
$$+i\theta\sigma^\mu\bar\theta\,\partial_\mu\phi_i^*(x)\phi_j(x) + \theta\sigma^\mu\bar\theta\,\partial_\mu\phi_i^*(x)\}\,\theta\sigma^\nu\bar\theta\,\partial_\nu\phi_j(x)\}$$
$$+\sqrt{2}\,i\theta\sigma^\mu\bar\theta\,\partial_\mu\phi_i^*(x)\}\;\theta\xi_j(x)\} - \frac{1}{4}\theta\theta\bar\theta\bar\theta\,\partial^\mu\partial_\mu\phi_i^*(x)\phi_j(x)$$
$$+\sqrt{2}\,\bar\theta\bar\xi_i(x)\phi_j(x) - \sqrt{2}\,i\bar\theta\bar\xi_i(x)\{\theta\sigma^\mu\bar\theta\,\partial_\mu\phi_j(x)\} + 2\bar\theta\bar\xi_i(x)\theta\xi_j(x)$$
$$+i\,\bar\theta\bar\xi_i(x)\{\theta\theta\,\partial_\mu\xi_j(x)\sigma^\mu\bar\theta\} + \sqrt{2}\,\bar\theta\bar\xi_i(x)\theta\theta F_j(x)$$
$$-\frac{i}{\sqrt{2}}\bar\theta\bar\theta\,\theta\sigma^\mu\partial_\mu\bar\xi_i(x)\phi_j(x) - i\bar\theta\bar\theta\,\theta\sigma^\mu\partial_\mu\bar\xi_i(x)\theta\xi_j(x)$$
$$+\bar\theta\bar\theta F_i^*(x)\phi_j(x) + \sqrt{2}\,\bar\theta\bar\theta F_i^*(x)\theta\xi_j(x) + \bar\theta\bar\theta F_i^*(x)\theta\theta F_j(x)$$

where —— : $\sim\theta^0,\bar\theta^0$, 〰 : $\sim\theta\theta\bar\theta$,

〰 : $\theta\sigma^\mu\bar\theta$, - - - - : $\bar\theta\theta\bar\theta$

〰 : $\theta\theta\bar\theta\bar\theta$

1) $\text{---} : \phi_i^* \phi_j(x) + \sqrt{2}\,\theta\,\xi_j(x)\,\phi_i^*(x) + \sqrt{2}\,\bar\theta\,\bar\xi_i(x)\,\phi_j(x) + \theta\theta\,\phi_i^*(x) F_j(x)$

$+ \bar\theta\bar\theta\, F_i^*(x)\phi_j(x) + 2\bar\theta\,\bar\xi_i(x)\,\theta\,\xi_j(x)$

2) $\sim\sim$ (I7a) $: -\frac{i}{\sqrt{2}}\theta\theta\bar\theta\bar\sigma^\mu \partial_\mu \xi_j(x)\,\phi_i^*(x) + \sqrt{2}\,\theta\theta\bar\theta\,\bar\xi_i(x) F_j(x)$

$+ \underbrace{\sqrt{2}\, i\,\theta\,\xi_j(x)\,\theta\sigma^\mu\bar\theta\,\partial_\mu\phi_i^*(x)}_{(*)}$

$(*) \overset{(I7e)}{=} \sqrt{2}\, i \left\{ -\frac{1}{2}\theta\theta\,\underbrace{\xi_j(x)\sigma^\mu\bar\theta}_{= -\bar\theta\bar\sigma^\mu\xi_j(x)\,(I7a)} + \underbrace{\theta\sigma^{\mu\nu}\theta}_{=0,\ (I7b)}\,\xi_j(x)\sigma_\nu\bar\theta \right\} \partial_\mu\phi_i^*(x)$

$= -\frac{i}{\sqrt{2}}\theta\theta\,\bar\theta_{\dot A}\,\bar\sigma^{\mu\dot A B}\,\partial_\mu \xi_{jB}(x)\,\phi_i^*(x) + \frac{i}{\sqrt{2}}\theta\theta\,\bar\theta_{\dot A}\,\bar\sigma^{\mu\dot A B}\,\xi_{jB}(x)\,\partial_\mu\phi_i^*(x)$

$+ \sqrt{2}\,\theta\theta\,\bar\theta_{\dot A}\,\bar\xi_i^{\dot A}(x)\,F_j(x)$

$= \sqrt{2}\,\theta\theta\,\bar\theta_{\dot A}\left\{ i\,\bar\sigma^{\mu\dot A B}\,\xi_{jB}(x)\,[\partial_\mu]\,\phi_i^*(x) + \bar\xi_i^{\dot A}(x)\,F_j(x)\right\}$

where $X\,[\partial_\mu]\,Y = \frac{1}{2}\left\{ X\,\partial_\mu Y - (\partial_\mu X)\,Y \right\}$

3) $\sim\sim : -i\,\theta\sigma^\mu\bar\theta\left\{\phi_i^*(x)\,\partial_\mu\phi_j(x)\right\} + i\,\theta\sigma^\mu\bar\theta\left\{\partial_\mu\phi_i^*(x)\,\phi_j(x)\right\}$

$= -2i\,\theta\sigma^\mu\bar\theta\left\{\phi_i^*(x)\,[\partial_\mu]\,\phi_j(x)\right\}$

4) $\text{- - - -} : \overbrace{-\sqrt{2}\, i\,\bar\theta\,\bar\xi_i(x)\,\theta\sigma^\mu\bar\theta\,\partial_\mu\phi_j(x)}^{(*)} - \frac{i}{\sqrt{2}}\bar\theta\bar\theta\,\theta\sigma^\mu\,\partial_\mu\bar\xi_i(x)\,\phi_j(x)$

$+ \sqrt{2}\,\bar\theta\bar\theta\,F_i^*(x)\,\theta\,\xi_j(x)$

$(*) = \sqrt{2}\, i\,\theta\,\bar\xi_i(x)\,\bar\theta\bar\sigma^\mu\theta\,\partial_\mu\phi_j(x)$

bard $\overset{(I7e)}{=} \sqrt{2}\, i \left\{ + \frac{1}{2}\bar\theta\bar\theta\,\underbrace{\bar\xi_i(x)\bar\sigma^\mu\theta}_{= -\theta\sigma^\mu\bar\xi_i(x)\,(I7a)} + \underbrace{\bar\theta\bar\sigma^{\mu\nu}\bar\theta}_{=0\ (I7.b)}\,\bar\xi_i(x)\bar\sigma_\nu\theta \right\} \partial_\mu\phi_j(x)$

$= \frac{i}{\sqrt{2}}\bar\theta\bar\theta\,\theta^A\sigma^\mu_{A\dot B}\,\bar\xi_i^{\dot B}(x)\,\partial_\mu\phi_j(x) - \frac{i}{\sqrt{2}}\bar\theta\bar\theta\,\theta^A\sigma^\mu_{A\dot B}\,\partial_\mu\bar\xi_i^{\dot B}(x)\,\phi_j(x)$

$+ \sqrt{2}\,\bar\theta\bar\theta\,\theta^A\,\xi_{jA}(x)\,F_i^*(x)$

$= \sqrt{2}\,\bar\theta\bar\theta\,\theta^A\left\{ i\,\sigma^\mu_{A\dot B}\,\bar\xi_i^{\dot B}(x)\,[\partial_\mu]\,\phi_j(x) + \xi_{jA}(x)\,F_i^*(x)\right\}$

5) $\underset{(I.7a)}{=} -\frac{1}{4}\phi_i^*(x)\,\theta\theta\bar\theta\bar\theta\,\partial^\mu\partial_\mu\phi_j(x) + \underbrace{\theta\sigma^\mu\bar\theta\,\partial_\mu\phi_i^*(x)\,\theta\sigma^\nu\bar\theta\,\partial_\nu\phi_j(x)}_{(*)}$

$- \frac{1}{4}\theta\theta\bar\theta\bar\theta\,\partial^\mu\partial_\mu\phi_i^*(x)\,\phi_j(x) \underbrace{- i\,\theta\theta\bar\theta\,\bar\xi_i(x)\,\bar\theta\,\bar\sigma^\mu\partial_\mu\xi_j(x)}_{(**)}$

$\underbrace{- i\,\bar\theta\bar\theta\,\theta\sigma^\mu\partial_\mu\bar\xi_i(x)\,\theta\xi_j(x)}_{(***)} + \bar\theta\bar\theta\theta\theta\,F_i^*(x)F_j(x)$

$(*) = \theta\sigma^\mu\bar\theta\,\theta\sigma^\nu\bar\theta\,\partial_\mu\phi_i^*(x)\partial_\nu\phi_j(x)$

$\overset{(I.7c)}{=} \frac{1}{2}g^{\mu\nu}\theta\theta\bar\theta\bar\theta\,\partial_\mu\phi_i^*(x)\partial_\nu\phi_j(x) = \frac{1}{2}\theta\theta\bar\theta\bar\theta\,\partial_\mu\phi_i^*(x)\partial^\mu\phi_j(x)$

$(**) \underset{\text{based}}{\overset{(I.7e)}{=}} -i\theta\theta\left\{-\frac{1}{2}\bar\theta\bar\theta\,\underbrace{\bar\xi_i(x)\bar\sigma^\mu\partial_\mu\xi_j(x)}_{=-\partial_\mu\xi_j(x)\sigma^\mu\bar\xi_i(x)} + \underbrace{\bar\theta\bar\sigma^{\mu\nu}\bar\theta}_{=0\ (I.7b)}\,\bar\xi_i(x)\bar\sigma_\nu\partial_\mu\xi_j(x)\right\}$

$= -\frac{i}{2}\theta\theta\bar\theta\bar\theta\,\partial_\mu\xi_j(x)\sigma^\mu\bar\xi_i(x)$

$(***) = -i\bar\theta\bar\theta\,\theta\xi_j(x)\,\theta\sigma^\mu\partial_\mu\bar\xi_i(x)$

$\overset{(I.7e)}{=} -i\bar\theta\bar\theta\left\{-\frac{1}{2}\theta\theta\,\xi_j(x)\sigma^\mu\partial_\mu\bar\xi_i(x) + \underbrace{\theta\sigma^{\mu\nu}\theta}_{=0\ (I.7b)}\,\xi_j(x)\sigma_\nu\partial_\mu\bar\xi_i(x)\right\}$

$= \frac{i}{2}\theta\theta\bar\theta\bar\theta\,\xi_j(x)\sigma^\mu\partial_\mu\bar\xi_i(x)$

$= \theta\theta\bar\theta\bar\theta\left\{-\frac{1}{4}\phi_i^*(x)\partial^\mu\partial_\mu\phi_j(x) - \frac{1}{4}\partial^\mu\partial_\mu\phi_i^*(x)\phi_j(x) + F_i^*(x)F_j(x)\right.$
$+ \frac{1}{2}\partial_\mu\phi_i^*(x)\partial^\mu\phi_j(x) - \frac{i}{2}\partial_\mu\xi_j(x)\sigma^\mu\bar\xi_i(x)$
$\left. + \frac{i}{2}\xi_j(x)\sigma^\mu\partial_\mu\bar\xi_i(x)\right\}$

$= \theta\theta\bar\theta\bar\theta\left\{F_i^*(x)F_j(x) + \frac{1}{2}\partial_\mu\phi_i^*(x)[\partial^\mu]\phi_j(x) - \frac{1}{2}\phi_i^*(x)[\partial_\mu]\partial^\mu\phi_j(x)\right.$
$\left. + i\,\xi_j(x)\sigma^\mu[\partial_\mu]\bar\xi_i(x)\right\}$

$[\partial_\mu]$ here on $(\xi_j\sigma^\mu)$ or only ξ_j ?

2) $V_{WZ}(x,\theta,\bar\theta) = \theta\sigma^\mu\bar\theta A_\mu(x) + \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\lambda(x)$
$+ \frac{1}{2}\theta\theta\bar\theta\bar\theta D(x)$

vector superfield in Wess-Zumino gauge

a)

Had $y^\mu = x^\mu - i\theta\sigma^\mu\bar\theta$, $\bar y^\mu = x^\mu + i\theta\sigma^\mu\bar\theta$

$\Leftrightarrow x^\mu = y^\mu + i\theta\sigma^\mu\bar\theta$, $x^\mu = \bar y^\mu - i\theta\sigma^\mu\bar\theta$

$\sim\theta^3, \bar\theta^3$ vanish

"+" for y^μ, "−" for $\bar y^\mu$

Actually $\partial_\mu^{(x)} A_\nu(x)\big|_{x=y}$?

$V_{WZ}(\overset{(-)}{y},\theta,\bar\theta) = \theta\sigma^\mu\bar\theta A_\mu(\overset{(-)}{y}) + \underbrace{\theta\sigma^\mu\bar\theta\,\partial_\nu A_\mu(\overset{(-)}{y})\{\pm i\theta\sigma^\nu\bar\theta\}}_{(*)}$
$+ \theta\theta\bar\theta\bar\lambda(\overset{(-)}{y}) + \cancel{\theta\theta\bar\theta\,\partial_\mu\bar\lambda(\overset{(-)}{y})\{\pm i\theta\sigma^\mu\bar\theta\}}$
$+ \bar\theta\bar\theta\theta\lambda(\overset{(-)}{y}) + \frac{1}{2}\theta\theta\bar\theta\bar\theta D(\overset{(-)}{y})$

$(*) = \pm i\partial_\nu A_\mu(\overset{(-)}{y})\,\theta\sigma^\mu\bar\theta\theta\sigma^\nu\bar\theta = \pm i\partial_\nu A_\mu(\overset{(-)}{y})\{\frac{1}{2}g^{\mu\nu}\theta\theta\bar\theta\bar\theta\}$
$= \pm\frac{i}{2}\partial^\mu A_\mu(\overset{(-)}{y})\,\theta\theta\bar\theta\bar\theta$

$= \theta\sigma^\mu\bar\theta A_\mu(\overset{(-)}{y}) \pm \frac{i}{2}\partial^\mu A_\mu(\overset{(-)}{y})\theta\theta\bar\theta\bar\theta + \theta\theta\bar\theta\bar\lambda(\overset{(-)}{y}) + \bar\theta\bar\theta\theta\lambda(\overset{(-)}{y})$
$+ \frac{1}{2}\theta\theta\bar\theta\bar\theta D(\overset{(-)}{y})$

$= \theta\sigma^\mu\bar\theta A_\mu(\overset{(-)}{y}) + \theta\theta\bar\theta\bar\lambda(\overset{(-)}{y}) + \bar\theta\bar\theta\theta\lambda(\overset{(-)}{y})$
$+ \frac{1}{2}\theta\theta\bar\theta\bar\theta\{D(\overset{(-)}{y}) \pm i\partial_\mu^{(y)} A^\mu(\overset{(-)}{y})\}$

b)

Had $\mathcal{D}_A^{(y)} = \partial_A - 2i\sigma^\mu_{A\dot B}\bar\theta^{\dot B}\partial_\mu^{(y)}$

$\bar{\mathcal{D}}_{\dot A}^{(\bar y)} = -\bar\partial_{\dot A} + 2i\theta^B\sigma^\mu_{B\dot A}\partial_\mu^{(\bar y)}$

$\partial_A(\theta\theta) = \partial_A(\theta^B\theta_B)$
$= \epsilon_{BC}\partial_A(\theta^B\theta^C) = \epsilon_{BC}\{\delta_A^B\theta^C - \theta^B\delta_A^C\}$
$= \epsilon_{AC}\theta^C + \epsilon_{AB}\theta^B = 2\theta_A$

Why not apply $\bar{\mathcal{D}}_{\dot A}^{(y)}$ as well?
$" -\bar\partial_{\dot A}"$
$\Rightarrow \bar{\mathcal{D}}_{\dot A}^{(\bar y)}$ not in $\bar y$ space, as $V(y,\ldots)$
everything in same space

Then $\mathcal{D}_A^{(y)} V_{WZ}(y,\theta,\bar\theta) = \sigma^\mu_{A\dot B}\bar\theta^{\dot B}A_\mu(y) \underbrace{- 2i\sigma^\nu_{A\dot B}\bar\theta^{\dot B}\theta\sigma^\mu\bar\theta\partial_\nu^{(y)}A_\mu(y)}_{(*)}$
$+ \frac{i}{2}\partial^{(y)\mu}A_\mu(y)\,2\theta_A\bar\theta\bar\theta + 2\theta_A\bar\theta\bar\lambda(y) - 2i\sigma^\nu_{A\dot B}\bar\theta^{\dot B}\theta\theta\bar\theta\partial_\nu^{(y)}\bar\lambda(y)$
$+ \bar\theta\bar\theta\lambda_A(y) + \theta_A\bar\theta\bar\theta D(y)$

$(*) = -2i(\sigma^\nu\bar\theta)_A\,\theta\sigma^\mu\bar\theta\,\partial_\nu^{(y)}A_\mu(y) = -2i\bar\theta\bar\theta\{\frac{1}{2}g^{\nu\mu}\theta_A - i(\sigma^{\nu\mu}\theta)_A\}\partial_\nu^{(y)}A_\mu(y)$
$= -i\bar\theta\bar\theta\theta_A\partial^{(y)\mu}A_\mu(y) - 2\bar\theta\bar\theta(\sigma^{\nu\mu}\theta)_A\partial_\nu^{(y)}A_\mu(y)$

$\stackrel{!}{=} \sigma^\mu_{A\dot B}\bar\theta^{\dot B}A_\mu(y) \cancel{- i\theta\bar\theta\bar\theta_A \partial^\mu A_\mu(y)} \overbrace{-2\bar\theta\bar\theta(\sigma^{\nu\mu}\theta)_A \partial^{(y)}_\nu A_\mu(y)}^{(*)}$

$\cancel{+ i\partial^\mu A_\mu(y)\theta_A\bar\theta\bar\theta} + 2\theta_A\bar\theta\bar\lambda(y) \underbrace{-2i\sigma^\nu_{A\dot B}\bar\theta^{\dot B}\theta\theta\bar\theta\partial^{(y)}_\nu\bar\lambda(y)}_{(**)} + \bar\theta\bar\theta\lambda_A(y) + \theta_A\bar\theta\bar\theta D(y)$

$(*) \stackrel{(I3)}{=} -2\bar\theta\bar\theta\sigma^{\mu\nu\,B}_{A}\theta_B\partial^{(y)}_\nu A_\mu(y) \stackrel{(I3)}{=} -\bar\theta\bar\theta\{\sigma^{\mu\nu\,B}_{A}\theta_B\partial^{(y)}_\nu A_\mu(y) - \sigma^{\nu\mu\,B}_{A}\theta_B\partial_\nu A_\mu(y)\}$

$= \bar\theta\bar\theta\{\sigma^{\mu\nu\,B}_{A}\theta_B\partial_\mu A_\nu(y) - \sigma^{\mu\nu\,B}_{A}\theta_B\partial_\nu A_\mu(y)\}$

$= \bar\theta\bar\theta\sigma^{\mu\nu\,B}_{A}\theta_B F_{\mu\nu}$

where $F_{\mu\nu} = \partial_\mu A_\nu(y) - \partial_\nu A_\mu(y)$

$(**) = -2i\sigma^\nu_{A\dot B}\bar\theta^{\dot B}\theta\theta\bar\theta_{\dot C}(\partial^{(y)}_\nu\bar\lambda(y))^{\dot C} \stackrel{(I5)}{=} 2i\sigma^\nu_{A\dot B}\theta\theta\bar\theta^{\dot B}\bar\theta^{\dot C}(\partial^{(y)}_\nu\bar\lambda(y))_{\dot C}$

$\stackrel{(I7f)}{=} i\sigma^\nu_{A\dot B}\theta\theta\epsilon^{\dot B\dot C}\bar\theta\bar\theta(\partial_\nu\bar\lambda(y))_{\dot C} = i\theta\theta\bar\theta\bar\theta\sigma^{\mu\dot C}_{A}(\partial_\mu\bar\lambda(y))_{\dot C}$

Can $\theta\theta$ just be pulled between σ and $\bar\theta$ here? ⇒ yes, anticomm. twice

$= \sigma^\mu_{A\dot B}\bar\theta^{\dot B}A_\mu(y) + 2\theta_A\bar\theta\bar\lambda(y) + \bar\theta\bar\theta\lambda_A(y)$

$+ \bar\theta\bar\theta\{\delta^B_A D(y) - \sigma^{\mu\nu\,B}_{A}F_{\mu\nu}\}\theta_B + i\theta\theta\bar\theta\bar\theta(\sigma^\mu\partial_\mu\bar\lambda(y))_A$

And $\bar D^{(\bar y)}_{\dot A} V_{WZ}(\bar y,\theta,\bar\theta) = (-\underset{= \frac{\partial}{\partial\bar\theta^{\dot A}}}{\bar\partial_{\dot A}} + 2i\theta^B\sigma^\mu_{B\dot A}\partial^{(\bar y)}_\mu)V_{WZ}(\bar y,\theta,\bar\theta)$

$\bar D^{(\bar y)}_{\dot A}$ here conj. of right chi. or of left chiral?

- $\theta\sigma^\mu\bar\theta \stackrel{(I7a)}{=} -\bar\theta\bar\sigma^\mu\theta = -\bar\theta_{\dot A}(\bar\sigma^\mu\theta)^{\dot A} \stackrel{(I5)}{=} \bar\theta^{\dot A}(\bar\sigma^\mu\theta)_{\dot A}$
- $\bar\partial_{\dot A}(\bar\theta\bar\theta) = \bar\partial_{\dot A}(\bar\theta_{\dot B}\bar\theta^{\dot B}) = \bar\partial_{\dot A}\epsilon_{\dot B\dot C}(\bar\theta^{\dot C}\bar\theta^{\dot B}) = \epsilon_{\dot B\dot C}\{\delta^{\dot C}_{\dot A}\bar\theta^{\dot B} - \bar\theta^{\dot C}\delta^{\dot B}_{\dot A}\}$

$= \{-\epsilon_{\dot A\dot B}\bar\theta^{\dot B} - \bar\theta^{\dot C}\epsilon_{\dot A\dot C}\} = -2\bar\theta_{\dot A}$

$= -(\bar\sigma^\mu\theta)_{\dot A}A_\mu(\bar y) \underbrace{+ 2i\theta^B\sigma^\nu_{B\dot A}\theta\sigma^\mu\bar\theta\partial^{(\bar y)}_\nu A_\mu(\bar y)}_{(*)}$

$+ \theta\theta\bar\lambda_{\dot A}(\bar y) + 2\bar\theta_{\dot A}\theta\lambda(\bar y) \underbrace{+ 2i\theta^B\sigma^\nu_{B\dot A}\bar\theta\bar\theta\theta\partial_\nu\lambda(\bar y)}_{(**)}$

$+ \theta\theta\bar\theta_{\dot A}\{D(\bar y) - i\partial^{(\bar y)}_\mu A^\mu(\bar y)\}$

∂_ν and θ^B

$(*) \underset{(I7a)}{\stackrel{(I7c)}{=}} 2i\partial^{(\bar y)}_\nu A_\mu(\bar y)(-\theta\theta)\{\tfrac12\bar\theta_{\dot A}g^{\nu\mu} + i(\bar\theta\bar\sigma^{\nu\mu})_{\dot A}\}$

$= i\partial^{(\bar y)\mu}A_\mu(\bar y)\theta\theta\bar\theta_{\dot A} - 2\partial^{(\bar y)}_\nu A_\mu(\bar y)(\bar\theta\bar\sigma^{\nu\mu})_{\dot A}\theta\theta$

?

$(**) = 2i\bar\theta\bar\theta\theta^B\sigma^\nu_{B\dot A}\theta^A(\partial_\nu\lambda(\bar y))_A$

$= 2i\bar\theta\bar\theta\sigma^\nu_{B\dot A}(-\tfrac12\epsilon^{BA}\theta\theta)(\partial_\nu\lambda(\bar y))_A = -i\bar\theta\bar\theta\theta\theta\sigma^\nu_{B\dot A}(\partial_\nu\lambda(\bar y))^B$

$= -i\bar\theta\bar\theta\theta\theta(\partial_\nu\lambda(\bar y)\sigma^\nu)_{\dot A}$

Can pull θ^B through $\sigma^\nu_{B\dot A}$ or rewrite ⇒ just a number here

$$\overset{!}{=} -(\bar\sigma^\mu\theta)_{\dot A} A_\mu(y) + i\cancel{\partial^{(y)\mu} A_\mu(y)\,\theta\theta\bar\theta_{\dot A}} \underbrace{- 2\partial_\nu^{(y)} A_\mu(y) (\bar\theta\bar\sigma^{\nu\mu})_{\dot A}\theta\theta}_{(*)}$$

$$+\theta\theta\bar\lambda_{\dot A}(y) + 2\bar\theta_{\dot A}\theta\lambda(y) - i\bar\theta\bar\theta\theta\theta(\partial_\nu\lambda(y)\sigma^\nu)_{\dot A}$$

$$+\theta\theta\bar\theta_{\dot A} D(y) \cancel{- i\theta\theta\bar\theta_{\dot A}\partial^{(y)}_\mu A^\mu(y)}$$

$$(*) = -2\theta\theta(\bar\theta\bar\sigma^{\nu\mu})_{\dot A}\partial_\nu^{(y)} A_\mu(y) = -\theta\theta(\bar\theta\bar\sigma^{\mu\nu})_{\dot A}\partial_\mu^{(y)} A_\nu(y)$$

$$-\theta\theta(\bar\theta\bar\sigma^{\nu\mu})_{\dot A}\partial_\nu^{(y)} A_\mu(y)$$

$$\overset{(I4)}{=} -\theta\theta(\bar\theta\bar\sigma^{\mu\nu})_{\dot A} F_{\mu\nu}(y)$$

where $F_{\mu\nu}(y) = \partial_\mu A_\nu(y) - \partial_\nu A_\mu(y)$

$$\overset{!}{=} -\bar\sigma^{\mu}{}_{\dot A}{}^{B}\theta_B A_\mu(y) + 2\bar\theta_{\dot A}\theta\lambda(y) + \theta\theta\bar\lambda_{\dot A}(y)$$

$$+\theta\theta\bar\theta_{\dot B}\{\delta^{\dot B}_{\dot A} D(y) - \bar\sigma^{\mu\nu\dot B}{}_{\dot A} F_{\mu\nu}(y)\}$$

$$-i\theta\theta\bar\theta\bar\theta\{\partial_\mu^{(y)}\lambda(y)\sigma^\mu\}_{\dot A}$$

c) Had $W_A = -\frac{1}{4}\bar D\bar D D_A V$, $\bar W_{\dot A} = -\frac{1}{4} DD\bar D_{\dot A} V$

$\bar D^{(y)}_{\dot B}$ or $\bar D^{(y)\dot B}$?

$\rightsquigarrow$ y-space $\rightsquigarrow W_A(y,\theta,\bar\theta) = -\frac{1}{4}\bar D^{(y)}_{\dot B}\bar D^{(y)\dot B} D^{(y)}_A V(y,\theta,\bar\theta)$

$$\bar D^{(y)}_{\dot B} = -\bar\partial_{\dot B} \Rightarrow \bar D^{(y)}_{\dot B}\bar D^{(y)\dot B} \underset{(I4c)}{=} -\bar\partial_{\dot B}\bar\partial^{\dot B} = -\frac{\partial}{\partial\bar\theta^{\dot B}}\frac{\partial}{\partial\bar\theta_{\dot B}}$$

$$\bar\partial_{\dot B}\bar\partial^{\dot B}\{\bar\theta\bar\theta\} = \bar\partial_{\dot B}\bar\partial^{\dot B}\{\bar\theta_{\dot C}\bar\theta^{\dot C}\} = \bar\partial_{\dot B}\bar\partial^{\dot B}\{\bar\theta_{\dot C}\bar\theta_{\dot D}\}\epsilon^{\dot C\dot D}$$

$$= \bar\partial_{\dot B}\{\delta^{\dot B}_{\dot C}\bar\theta_{\dot D} - \bar\theta_{\dot C}\delta^{\dot B}_{\dot D}\}\epsilon^{\dot C\dot D} = \{\delta^{\dot B}_{\dot C}\epsilon_{\dot D\dot E}\delta^{\dot E}_{\dot B} - \epsilon_{\dot C\dot E}\delta^{\dot E}_{\dot B}\delta^{\dot B}_{\dot D}\}\epsilon^{\dot C\dot D}$$

$$= -\epsilon^{\dot B\dot D}\epsilon_{\dot B\dot D} - \epsilon_{\dot B\dot C}\epsilon^{\dot B\dot C} = 2\epsilon^{\dot B\dot D}\epsilon_{\dot D\dot B} = 4$$

$$= -\frac{1}{4}(-4\lambda_A(y) - 4D(y)\theta_A + 4\sigma^{\mu\nu}{}_A{}^B F_{\mu\nu}\theta_B - 4i\theta\theta(\sigma^\mu\partial_\mu\bar\lambda(y))_A)$$

$$= \lambda_A(y) + D(y)\theta_A - (\sigma^{\mu\nu}\theta)_A F_{\mu\nu}(y) + i\theta\theta\sigma^\mu_{A\dot B}\partial^{(y)}_\mu\bar\lambda^{\dot B}(y)$$

And $\bar W_{\dot A}(\bar y,\theta,\bar\theta) = -\frac{1}{4} D^{(\bar y)B} D^{(\bar y)}_B \bar D_{\dot A} V(\bar y,\theta,\bar\theta)$

$$D^{(\bar y)}_B = \partial_B \rightarrow D^B D_B = -\partial^B\partial_B$$

$$\partial\partial(\theta\theta) = 4$$

$$= -\frac{1}{4}\{-4\bar\lambda_{\dot A}(\bar y) - 4\bar\theta_{\dot A} D(\bar y) + 4\bar\theta_{\dot B}\bar\sigma^{\mu\nu\dot B}{}_{\dot A} F_{\mu\nu}(\bar y) + 4i\bar\theta\bar\theta(\partial^{(\bar y)}_\mu\lambda(\bar y)\sigma^\mu)_{\dot A}$$

$$= \bar{\lambda}_{\dot{A}}(y) + D(y)\,\bar{\theta}_{\dot{A}} - (\bar{\theta}\bar{\sigma}^{\mu\nu})_{\dot{A}} F_{\mu\nu}(y) - i\theta\bar{\theta}(\partial_\mu \lambda(y)\sigma^\mu)_{\dot{A}}$$

$$= \bar{\lambda}_{\dot{A}}(y) + D(y)\,\bar{\theta}_{\dot{A}} - \epsilon_{\dot{A}\dot{B}}(\bar{\sigma}^{\mu\nu}\bar{\theta})^{\dot{B}} F_{\mu\nu}(y) - i\theta\bar{\theta}(\partial_\mu \lambda(y)\sigma^\mu)_{\dot{A}}$$? last step?

d) We are looking for the $\theta\theta$ component of $W^A W_A$

$$W^A W_A = \epsilon^{AB} W_B W_A$$

$$= \epsilon^{AB}\{\lambda_B(y) + D(y)\theta_B - (\sigma^{\mu\nu}\theta)_B F_{\mu\nu}(y) + i\theta\theta\sigma^\mu_{B\dot{C}}\partial^{(y)}_\mu \bar{\lambda}^{\dot{C}}(y)\}$$

$$\times\{\lambda_A(y) + D(y)\theta_A - (\sigma^{\kappa\lambda}\theta)_A F_{\kappa\lambda}(y) + i\theta\theta\sigma^k_{A\dot{D}}\partial^{(y)}_k \bar{\lambda}^{\dot{D}}(y)\}$$

$\theta\theta$-terms

$$= \underbrace{i\epsilon^{AB}\lambda_B(y)\theta\theta\sigma^k_{A\dot{D}}\partial_k\bar{\lambda}^{\dot{D}}(y)}_{i\theta\theta(\lambda(y)\sigma^\mu\partial_\mu\bar{\lambda}(y))} + \underbrace{\epsilon^{AB}D(y)\theta_B D(y)\theta_A}_{\theta\theta D^2(y)}$$

$$\underbrace{-\epsilon^{AB}D(y)\theta_B(\sigma^{\kappa\lambda}\theta)_A F_{\kappa\lambda}(y)}_{-D(y)(\theta\sigma^{\kappa\lambda}\theta)F_{\kappa\lambda}(y)} \underbrace{-\epsilon^{AB}(\sigma^{\mu\nu}\theta)_B F_{\mu\nu}(y)D(y)\theta_A}_{-D(y)\underbrace{(\theta\sigma^{\mu\nu}\theta)}_{=0}F_{\mu\nu}(y)} \quad (\{\theta_A,\theta_B\}=0)$$

$$+\epsilon^{AB}(\sigma^{\mu\nu}\theta)_B F_{\mu\nu}(y)(\sigma^{\kappa\lambda}\theta)_A F_{\kappa\lambda}(y)$$

$$\underbrace{+i\epsilon^{AB}\theta\theta\sigma^\mu_{B\dot{C}}\partial^{(y)}_\mu\bar{\lambda}^{\dot{C}}(y)\lambda_A(y)}$$

$$\overset{\{\lambda_A,\lambda^{\dot{C}}\}=0}{\Rightarrow} i\theta\theta\epsilon^{BA}\lambda_A(y)\sigma^\mu_{B\dot{C}}\partial_\mu\bar{\lambda}^{\dot{C}}(y)$$

$$= i\theta\theta\lambda(y)\sigma^\mu\partial_\mu\bar{\lambda}(y)$$

$$= \theta\theta\{D^2(y) + 2i\lambda(y)\sigma^\mu\partial^{(y)}_\mu\bar{\lambda}(y)\}$$

$$\underbrace{+\epsilon^{AB}F_{\mu\nu}(y)F_{\kappa\lambda}(y)\sigma^{\mu\nu}{}_B{}^C\theta_C\sigma^{\kappa\lambda}{}_A{}^D\theta_D}_{(*)}$$

$$(*) = \frac{1}{2}\epsilon^{AB}\epsilon_{CD}F_{\mu\nu}(y)F_{\kappa\lambda}(y)\sigma^{\mu\nu}{}_B{}^C\sigma^{\kappa\lambda}{}_A{}^D\theta\theta$$

Why index down for σ^μ: $(\sigma^\mu\bar{\sigma}^\nu)_A{}^B = \sigma^\mu_{A\dot{C}}\bar{\sigma}^{\nu\dot{C}B}$?

$$\sigma^{\mu\nu}{}_A{}^B = \frac{i}{4}\{(\sigma^\mu\bar{\sigma}^\nu)_A{}^B - (\mu\leftrightarrow\nu)\}$$

$$= \frac{i}{4}\{\sigma^\mu_{A\dot{C}}\bar{\sigma}^{\nu\dot{C}B} - (\mu\leftrightarrow\nu)\}$$

$$= \frac{i}{4}\{\epsilon_{AD}\epsilon_{\dot{C}\dot{E}}\bar{\sigma}^{\mu\dot{E}D}\epsilon^{\dot{C}\dot{F}}\epsilon^{BG}\sigma^\nu_{G\dot{F}} - (\mu\leftrightarrow\nu)\}$$

$$= \frac{i}{4}\{-\delta_{\dot{E}}{}^{\dot{F}}\epsilon_{AD}\epsilon^{BG}\bar{\sigma}^{\mu\dot{E}D}\sigma^\nu_{G\dot{F}} - (\mu\leftrightarrow\nu)\}$$

$$= \frac{i}{4}\{\epsilon_{DA}\epsilon^{BG}\bar{\sigma}^{\mu\dot{E}D}\sigma^\nu_{G\dot{E}} - (\mu\leftrightarrow\nu)\}$$

Now pulling indices differently?

$$= \frac{i}{4}\{(\sigma^\nu\bar{\sigma}^\mu)^B{}_A - (\mu\leftrightarrow\nu)\} = -\sigma^{\mu\nu B}{}_A$$

$$= -\frac{1}{2}\theta\theta\epsilon^{AB}\epsilon_{CD}F_{\mu\nu}(y)F_{\kappa\lambda}(y)\sigma^{\mu\nu}{}_B{}^C\sigma^{\kappa\lambda D}{}_A$$

$$= -\frac{1}{2}\theta\theta F_{\mu\nu}(y)F_{\kappa\lambda}(y)\,\mathrm{tr}(\sigma^{\mu\nu}\sigma^{\kappa\lambda})$$

$$= -\frac{1}{2}\theta\theta F_{\mu\nu}(y) F_{\kappa\lambda}(y) \frac{1}{2}\{ g^{\mu\kappa} g^{\nu\lambda} - g^{\mu\lambda} g^{\nu\kappa} + i\epsilon^{\mu\nu\kappa\lambda} \}$$

$$= -\frac{1}{4}\theta\theta \{ F_{\mu\nu}(y) F^{\mu\nu}(y) - F_{\mu\nu}(y) F^{\nu\mu}(y) + i\epsilon^{\mu\nu\kappa\lambda} F_{\mu\nu}(y) F_{\kappa\lambda}(y) \}$$

$$= -\frac{1}{2}\theta\theta F_{\mu\nu}(y) F^{\mu\nu}(y) - \frac{i}{4}\theta\theta \epsilon_{\mu\nu\kappa\lambda} F^{\kappa\lambda}(y) F^{\mu\nu}(y)$$

$$= -\frac{1}{2}\theta\theta F_{\mu\nu}(y) F^{\mu\nu}(y) - \frac{i}{2}\theta\theta \hat{F}_{\mu\nu}(y) F^{\mu\nu}(y)$$

where $\hat{F}_{\mu\nu}(y) = \frac{1}{2}\epsilon_{\mu\nu\kappa\lambda} F^{\kappa\lambda}(y)$

$$= \theta\theta \{ D^2(y) + 2i\lambda(y)\sigma^\mu \partial_\mu^{(y)} \bar{\lambda}(y) - \frac{1}{2} F_{\mu\nu}(y) F^{\mu\nu}(y) - \frac{i}{2} \hat{F}_{\mu\nu}(y) F^{\mu\nu}(y) \}$$

WRONG?!

d) We are looking for the $\theta\theta$ component of $W^A W_A$

What's the hint good for??

$$W^A W_A = \epsilon^{AB} W_B W_A$$

$$= \epsilon^{AB} \{ \lambda_B(y) + D(y)\theta_B - (\sigma^{\mu\nu}\theta)_B F_{\mu\nu}(y) + i\theta\theta\sigma^\mu_{B\dot{C}} \partial^{(y)}_\mu \bar{\lambda}^{\dot{C}}(y) \}$$
$$\times \{ \lambda_A(y) + D(y)\theta_A - (\sigma^{\kappa\lambda}\theta)_A F_{\kappa\lambda}(y) + i\theta\theta\sigma^\kappa_{A\dot{D}} \partial^{(y)}_\kappa \bar{\lambda}^{\dot{D}}(y) \}$$

$\theta\theta$ - terms

$$= \underbrace{i\lambda_B(y)\theta\theta(\sigma^\kappa \partial^{(y)}_\kappa \bar{\lambda}(y))_A \epsilon^{AB}}_{=+i\theta\theta\lambda(y)\sigma^\kappa\partial_\kappa\bar{\lambda}(y)} \underbrace{- (\sigma^{\mu\nu}\theta)_B F_{\mu\nu}(y) D(y)\theta_A \epsilon^{AB}}_{\theta\sigma^{\mu\nu}\theta\, F_{\mu\nu}(y) D(y)}$$
$$+ \underbrace{(\sigma^{\mu\nu}\theta)_B F_{\mu\nu}(y) (\sigma^{\kappa\lambda}\theta)_A F_{\kappa\lambda}(y)\epsilon^{AB}}_{(*)}$$
$$+ \underbrace{D(y)\theta_B D(y)\theta_A \epsilon^{AB}}_{D(y)D(y)\theta\theta} \underbrace{- D(y)\theta_B(\sigma^{\kappa\lambda}\theta)_A F_{\kappa\lambda}(y)\epsilon^{AB}}_{-D(y)(\theta\sigma^{\kappa\lambda}\theta)F_{\kappa\lambda}(y)}$$

Cancel

$$+ \underline{i\theta\theta\sigma^\mu_{B\dot{C}} \partial^{(y)}_\mu \bar{\lambda}^{\dot{C}}(y) \lambda_A(y) \epsilon^{AB}}$$
$$- i\theta\theta(\lambda(y)\sigma^\mu \partial_\mu \bar{\lambda}(y))$$

$$(*) = (\sigma^{\mu\nu}\theta)^A (\sigma^{\kappa\lambda}\theta)_A F_{\mu\nu}(y) F_{\kappa\lambda}(y)$$
$$= (\sigma^{\mu\nu}\theta)^A (-\theta\sigma^{\kappa\lambda})_A F_{\mu\nu}(y) F_{\kappa\lambda}(y)$$
$$= -\sigma^{\mu\nu AB}\theta_B \theta^C \sigma^{\kappa\lambda}{}_{CA} F_{\mu\nu}(y) F_{\kappa\lambda}(y)$$
$$= -\epsilon^{CD}\sigma^{\mu\nu AB}\tfrac{1}{2}\epsilon_{BD}\theta\theta\sigma^{\kappa\lambda}{}_{CA} F_{\mu\nu}(y) F_{\kappa\lambda}(y)$$
$$= \tfrac{1}{2}\delta_B{}^C \sigma^{\mu\nu AB}\theta\theta\sigma^{\kappa\lambda}{}_{CA} F_{\mu\nu}(y) F_{\kappa\lambda}(y)$$
$$= \tfrac{1}{2}\,\mathrm{tr}(\sigma^{\mu\nu}\sigma^{\kappa\lambda}) F_{\mu\nu}(y) F_{\kappa\lambda}(y)\theta\theta$$
$$= \tfrac{1}{4}\theta\theta(g^{\mu\kappa}g^{\nu\lambda} - g^{\mu\lambda}g^{\nu\kappa} + i\epsilon^{\mu\nu\kappa\lambda}) F_{\mu\nu}(y) F_{\kappa\lambda}(y)$$
$$= \tfrac{1}{4}\theta\theta\{ F_{\mu\nu}(y)F^{\mu\nu}(y) - F_{\mu\nu}(y)F^{\nu\mu}(y) + i\epsilon^{\mu\nu\kappa\lambda} F_{\mu\nu}(y) F_{\kappa\lambda}(y) \}$$
$$= \tfrac{1}{2}\theta\theta F_{\mu\nu}(y)F^{\mu\nu}(y) + \tfrac{i}{4}\theta\theta\epsilon^{\mu\nu\kappa\lambda}$$

$$= \theta\theta \}$$