Disclaimer

The solution at hand was written in the course of the respective class at the University of Bonn. If not stated differently on top of the first page or the following website, the solution was prepared and handed in solely by me, Marvin Zanke. Anything in a different color than the ball pen blue is usually a correction that I or a tutor made. For more information and all my material, check:
https://www.physics-and-stuff.com/

I raise no claim to correctness and completeness of the given solutions! This equally applies to the corrections mentioned above.

10.07.2018 1) a) The most general SUSY and U(1) gauge invariant Lagrangian describing the interactions of an abelian vector SF V and left-ch. SFs Φ_+, Φ_- is given by

$$\mathcal{L} = \int d^4\theta \left\{ \Phi_i^\dagger e^{2gt_iV} \Phi_i + 2\eta V \right\} + \frac{1}{4}\left\{ \int d^2\theta\, W^A W_A + \int d^2\bar\theta\, \bar W_{\dot A} \bar W^{\dot A} \right\}$$

$$+ \int d^2\theta \left\{ W(\Phi_i) + h.c. \right\}$$

why e^{+2qV} and not e^{-2qV}? Trafo w/ (-)? ⇝ yes if SO charge $+q$ ✓

$$= \int d^4\theta \left\{ \Phi_+^\dagger e^{2qV} \Phi_+ + \Phi_-^\dagger e^{-2qV} \Phi_- \right\} + \frac{1}{4}\left\{ \int d^2\theta\, W^A W_A + \int d^2\bar\theta\, \bar W_{\dot A} \bar W^{\dot A} \right\}$$

$$+ \int d^2\theta\, M \left\{ \Phi_+ \Phi_- + \Phi_+^\dagger \Phi_-^\dagger \right\}$$

where $W(\Phi) = h_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{6} f_{ijk} \Phi_i \Phi_j \Phi_k$

gauge sym. or inv.?

has to respect gauge invariance ⇝ $2\times \Phi$'s w/ opposite charge

$\longrightarrow W(\Phi_+, \Phi_-) = M \Phi_+ \Phi_-$

and $\Phi_\pm(y,\theta) = \varphi_\pm(y) + \sqrt{2}\,\theta \xi_\pm(y) + \theta\theta F_\pm(y)$

w/ opposite charges $q, -q$

$$\Phi_+ \longrightarrow \Phi_+' = e^{-2iq\Lambda} \Phi_+, \quad \Phi_- \longrightarrow \Phi_-' = e^{2iq\Lambda} \Phi_-$$

$$V \to V' = V + i\Lambda - i\Lambda^\dagger$$

Also we set $\eta = 0$, i.e. vanishing Fayet-Iliopoulos term

We expand the exponential $e^{\pm 2qV}$ and work in WZ-gauge, where $V^3 = 0$

$$⇝ \Phi_\pm^\dagger e^{\pm 2qV} \Phi_\pm = \Phi_\pm^\dagger \left\{ 1 \pm 2qV + 2q^2 V^2 \right\} \Phi_\pm$$

Inserting relations found in the lecture for $\Phi_i^\dagger \Phi_i$, $\Phi_i^\dagger V \Phi_i$, $\Phi_i^\dagger V^2 \Phi_i$, $W^A W_A$, $\bar W_{\dot A} \bar W^{\dot A}$, $\Phi_i \Phi_j$, we finally find

$$\mathcal{L} = \{F_+^* F_+ + \partial_\mu \phi_+^* \partial^\mu \phi_+ + i\xi_+ \sigma^\mu [\partial_\mu] \bar\xi_+\}$$
$$+ q\{D\phi_+^*\phi_+ - 2iA^\mu \phi_+^* [\partial_\mu]\phi_+ - \bar\xi_+ \bar\sigma^\mu \xi_+ A_\mu$$
$$- \sqrt{2} \phi_+ \bar\lambda \bar\xi_+ - \sqrt{2} \phi_+^* \lambda \xi_+\}$$
$$+q^2\{A^\mu A_\mu \phi_+^*\phi_+\}$$
$$+\{F_-^* F_- + \partial_\mu\phi_-^*\partial^\mu\phi_- + i\xi_-\sigma^\mu[\partial_\mu]\bar\xi_-\}$$
$$-q\{D\phi_-^*\phi_- - 2iA^\mu\phi_-^*[\partial_\mu]\phi_- - \bar\xi_-\bar\sigma^\mu\xi_- A_\mu - \sqrt{2}\phi_-\bar\lambda\bar\xi_- - \sqrt{2}\phi_-^*\lambda\xi_-\}$$
$$+q^2\{A^\mu A_\mu\phi_-^*\phi_-\}$$
$$+\frac{1}{4}\{2D^2 + 2i\lambda\sigma^\mu\partial_\mu\bar\lambda - 2i(\partial_\mu\lambda)\sigma^\mu\bar\lambda - F_{\mu\nu}F^{\mu\nu}\}$$
$$+M\{\phi_+F_- + \phi_-F_+ - \xi_+\xi_- + \phi_+^*F_-^* + \phi_-^*F_+^* - \bar\xi_+\bar\xi_-\}$$

✓ In book t_i between ϕ_i^* and ϕ_i, why ⇝ doesn't matter

b) D and $F_\pm^{(*)}$ don't propagate (→ no derivatives of these fields in $\mathcal{L}$) and are thus auxiliary fields that can be integrated out:

$$0 = \frac{\partial \mathcal{L}}{\partial D} = D + q(|\phi_+|^2 - |\phi_-|^2)$$
$$\rightsquigarrow D = -q\{|\phi_+|^2 - |\phi_-|^2\}$$

$$0 = \frac{\partial \mathcal{L}}{\partial F_+} = F_+^* + M\phi_- \rightsquigarrow F_+^* = -M\phi_-$$
$$F_+ = -M\phi_-^*$$

$$0 = \frac{\partial \mathcal{L}}{\partial F_-} = F_-^* + M\phi_+ \rightsquigarrow F_-^* = -M\phi_+$$
$$F_- = -M\phi_+^*$$

up to tot. derivatives
$X[\partial_\mu]Y = \frac{1}{2}(X\partial_\mu Y - (\partial_\mu X)Y)$
$= X\partial_\mu Y - \frac{1}{2}\partial_\mu(XY)$

$$\mathcal{L} = M^2|\phi_-|^2 + \partial_\mu\phi_+^*\partial^\mu\phi_+ + i\xi_+\sigma^\mu\partial_\mu\bar\xi_+ - q^2|\phi_+|^2\{|\phi_+|^2 - |\phi_-|^2\}$$
$$-2iqA^\mu\phi_+^*[\partial_\mu]\phi_+ + q\xi_+\sigma^\mu\bar\xi_+A_\mu - \sqrt{2}q\phi_+\bar\lambda\bar\xi_+ - \sqrt{2}q\phi_+^*\lambda\xi_+$$
$$+q^2|\phi_+|^2A^\mu A_\mu + M^2|\phi_+|^2 + \partial_\mu\phi_-^*\partial^\mu\phi_- + i\xi_-\sigma^\mu\partial_\mu\bar\xi_-$$
$$+q^2|\phi_-|^2\{|\phi_+|^2 - |\phi_-|^2\} + 2iqA^\mu\phi_-^*[\partial_\mu]\phi_- - q\xi_-\sigma^\mu\bar\xi_-A_\mu$$
$$+\sqrt{2}q\phi_-\bar\lambda\bar\xi_- + \sqrt{2}q\phi_-^*\lambda\xi_- + q^2|\phi_-|^2A^\mu A_\mu + \frac{q^2}{2}\{|\phi_+|^2 - |\phi_-|^2\}$$
$$+i\lambda\sigma^\mu\partial_\mu\bar\lambda - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - M^2|\phi_+|^2 - M^2|\phi_-|^2 - M\xi_+\xi_-$$
$$-M^2|\phi_+|^2 - M^2|\phi_-|^2 - M\bar\xi_+\bar\xi_-$$

where we dropped total derivatives of the form e.g.

$$\frac{1}{4}\{2i\lambda\sigma^\mu\partial_\mu\bar\lambda - 2i(\partial_\mu\lambda)\sigma^\mu\bar\lambda\} = \frac{i}{2}\lambda\sigma^\mu\partial_\mu\bar\lambda - \frac{i}{2}\partial_\mu\{\lambda\sigma^\mu\bar\lambda\} + \frac{i}{2}\lambda\sigma^\mu\partial_\mu\bar\lambda$$

$$= i\lambda\sigma^\mu\partial_\mu\bar\lambda + \text{tot. der.}$$

This can further be simplified to:

In book they kept D and F instead of replacing it by eq. of constraint? ⇝ possible but not final result

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\lambda\sigma^\mu\partial_\mu\bar\lambda + \overbrace{i\xi_+\sigma^\mu\partial_\mu\bar\xi_+ + i\xi_-\sigma^\mu\partial_\mu\bar\xi_-}^{+q\xi_+\sigma^\mu\bar\xi_+A_\mu - q\xi_-\sigma^\mu\bar\xi_-A_\mu}$$
$$+ \partial_\mu\phi_+^*\partial^\mu\phi_+ + q^2|\phi_+|^2A^\mu A_\mu - 2iqA^\mu\phi_+^*[\partial_\mu]\phi_+$$
$$+ \partial_\mu\phi_-^*\partial^\mu\phi_- + q^2|\phi_-|^2A^\mu A_\mu + 2iqA^\mu\phi_-^*[\partial_\mu]\phi_-$$
$$- M^2\{|\phi_+|^2 + |\phi_-|^2\} - M\{\xi_+\xi_- + \bar\xi_+\bar\xi_-\}$$
$$- \frac{q^2}{2}\{|\phi_+|^2 - |\phi_-|^2\}^2 - \sqrt{2}q\{\bar\lambda(\bar\xi_+\phi_+ - \bar\xi_-\phi_-) + \lambda(\xi_+\phi_+^* - \xi_-\phi_-^*)\}$$

Introducing a gauge covariant derivative $\Delta_\mu = \partial_\mu + iqA_\mu \Rightarrow \Delta_\mu^\dagger = \partial_\mu - iqA_\mu$ one finds

In book order changed between ∂_μ and $[\partial_\mu]$? ⇝ yes

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \underbrace{i\lambda\sigma^\mu\partial_\mu\bar\lambda}_{(1)} + \underbrace{i\xi_+\sigma^\mu\Delta_\mu^\dagger\bar\xi_+ + i\xi_-\sigma^\mu\Delta_\mu\bar\xi_-}_{(2)}$$
$$+ |\Delta_\mu\phi_+|^2 + |\Delta_\mu^\dagger\phi_-|^2 - M^2\{|\phi_+|^2 + |\phi_-|^2\}$$
$$- M\underbrace{\{\xi_+\xi_- + \bar\xi_+\bar\xi_-\}}_{(3)} - \frac{q^2}{2}\{|\phi_+|^2 - |\phi_-|^2\}^2$$
$$- \sqrt{2}q\{\underbrace{\bar\lambda(\bar\xi_+\phi_+ - \bar\xi_-\phi_-)}_{(4)} + \underbrace{\lambda(\xi_+\phi_+^* - \xi_-\phi_-^*)}_{(5)}\}$$

As the mixed terms from $|\Delta_\mu\phi_+|^2$, $|\Delta_\mu^\dagger\phi_-|^2$ are

$$|\Delta_\mu\phi_+|^2 = (\partial_\mu + iqA_\mu)\phi_+(\partial_\mu - iqA_\mu)\phi_+^*$$
$$\supset \partial_\mu\phi_+(-iqA_\mu)\phi_+^* + iqA_\mu\phi_+\partial\phi_+^*$$
$$|\Delta_\mu^\dagger\phi_-|^2 = (\partial_\mu - iqA_\mu)\phi_-(\partial + iqA_\mu)\phi_-^*$$
$$\supset \partial_\mu\phi_-(iqA_\mu)\phi_-^* - iqA_\mu\phi_-\partial\phi_-^*$$

$\bar\Psi \hat= \Psi^\dagger\gamma^0$? ⇝ we will! note the change of order

c) Defining the Dirac fermion $\Psi_a = \begin{pmatrix}(\xi_+)_A \\ (\bar\xi_-)^{\dot A}\end{pmatrix}$, $\bar\Psi_b = ((\xi_-)^B, (\bar\xi_+)_{\dot B})$ $\hat= \Psi$

Why first comp. index down?

$$\lambda_a = \begin{pmatrix}\lambda_A \\ \bar\lambda^{\dot A}\end{pmatrix},\quad \bar\lambda_b = (\lambda^B, \bar\lambda_{\dot B})$$
$$\hat= \lambda_M$$

We take a look at the different underlined terms

(1) $i\lambda\sigma^\mu\partial_\mu\bar\lambda = \frac{i}{2}\{\lambda\sigma^\mu\partial_\mu\bar\lambda - \partial_\mu\lambda\sigma^\mu\bar\lambda\}$ + tot. der.

$= \frac{i}{2}\{\lambda\sigma^\mu\partial_\mu\bar\lambda + \bar\lambda\bar\sigma^\mu\partial_\mu\lambda\}$

$\overset{(I.13c)}{=} \frac{i}{2}\{\overline{\lambda_M}\gamma^\mu\partial\lambda_M\}$

$\Psi \hat{=} \Psi_a \hat{=} \Psi_b$

(2) $i\{\xi_+\sigma^\mu\Delta_\mu^+\bar\xi_+ + \xi_-\sigma^\mu\Delta_\mu\bar\xi_-\}$

$\xi_+\sigma^\mu(\partial_\mu - iqA_\mu)\bar\xi_+ = -\partial_\mu\xi_+\sigma^\mu\bar\xi_+ - iqA_\mu\xi_+\sigma^\mu\bar\xi_+$

up to total der.

$= \bar\xi_+\bar\sigma^\mu\partial_\mu\xi_+ + iqA_\mu\bar\xi_+\bar\sigma^\mu\xi_+ = \bar\xi_+\bar\sigma^\mu\Delta_\mu\xi_+$

$= i\{\bar\xi_+\bar\sigma^\mu\Delta_\mu\xi_+ + \xi_-\sigma^\mu\Delta_\mu\bar\xi_-\} \overset{(I.12c)}{=} i\bar\Psi\gamma^\mu\Delta_\mu\Psi$

(3) $\xi_+\xi_- + \bar\xi_+\bar\xi_- = \xi_-\xi_+ + \bar\xi_+\bar\xi_- \overset{(I.14a)}{=} \bar\Psi\Psi$

(4) $\bar\lambda\bar\xi_+\phi_+ - \bar\lambda\bar\xi_-\phi_- = \bar\Psi_L\lambda_M\phi_+ - \overline{\lambda_M}\Psi_R\phi_-$

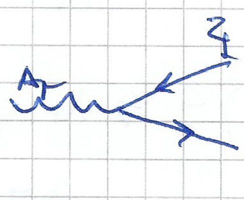
(5) $\lambda\xi_+\phi_+^* - \lambda\xi_-\phi_-^* = \overline{\lambda_M}\Psi_L\phi_+^* - \bar\Psi_R\lambda_M\phi_-^*$

$$\Rightarrow \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\{\overline{\lambda_M}\gamma^\mu\partial_\mu\lambda_M\} + \underbrace{i\bar\Psi\gamma^\mu\Delta_\mu\Psi}_{(1)}$$
$$+ \underbrace{|\Delta_\mu\phi_+|^2}_{(2)} + \underbrace{|\Delta_\mu^+\phi_-|^2}_{(3)} - M^2\{|\phi_+|^2 + |\phi_-|^2\}$$
$$- M\bar\Psi\Psi - \underbrace{\frac{q^2}{2}\{|\phi_+|^2 - |\phi_-|^2\}^2}_{(4)}$$
$$\underbrace{-\sqrt{2}q\{\bar\Psi_L\lambda_M\phi_+ - \overline{\lambda_M}\Psi_R\phi_- + \overline{\lambda_M}\Psi_L\phi_+^* - \bar\Psi_R\lambda_M\phi_-^*\}}_{(5)}$$

✓ How to see from $\mathcal{L}$ that it's a Majorana particle? → factor 1/2 (and also no mass here, as gaugino to massless gauge boson photon)

d) To read off the Feynman rules for all 3- and 4-point vertices, we consider the part of the Lagrangian w/ 3 or 4 fields:

① $i\bar{\psi}_f \gamma^\mu D_\mu \psi_f \supset -q\bar{\psi}_f \gamma^\mu A_\mu \psi_f$ ⇝



$: -iq\gamma^\mu$

② $|D_\mu \phi_+|^2 = (\partial_\mu + iqA_\mu)\phi_+ (\partial^\mu - iqA^\mu)\phi_+^*$

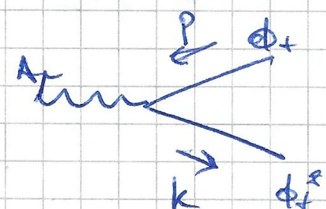
$\supset iqA_\mu \phi_+ \partial^\mu \phi_+^* - iqA_\mu \phi_+^* \partial^\mu \phi_+ + q^2 A^\mu A_\mu \phi_+ \phi_+^*$

$= iqA_\mu \{\phi_+ \partial^\mu \phi_+^* - \phi_+^* \partial^\mu \phi_+\} + q^2 A_\mu A^\mu \phi_+ \phi_+^*$

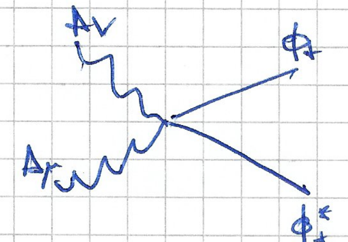
Could also write those 2 as 2 different rules instead of combining them?

If do an int. by parts, get different Feynman rules?

⇝



$: -iq\{k^\mu + p^\mu\}$



$: 2iq^2 g_{\mu\nu}$

✓

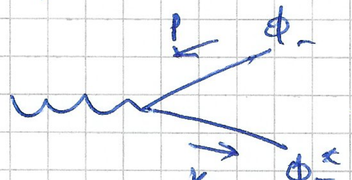
What is ϕ_+^*? $\in \phi_-$? Needed for charge cons.? ⇝ outgoing ϕ_+

③ $|D_\mu^\dagger \phi_-| = (\partial_\mu - iqA_\mu)\phi_- (\partial^\mu + iqA^\mu)\phi_-^*$

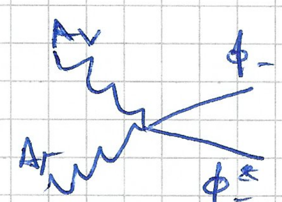
$\supset -iqA_\mu \phi_- \partial^\mu \phi_-^* + iqA_\mu \phi_-^* \partial^\mu \phi_- + q^2 A_\mu A^\mu \phi_- \phi_-^*$

$= iqA_\mu \{\phi_-^* \partial^\mu \phi_- - \phi_- \partial^\mu \phi_-^*\} + q^2 A_\mu A^\mu \phi_- \phi_-^*$

⇝

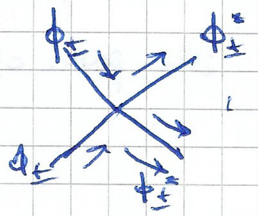


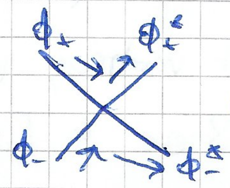
$: iq\{k^\mu + p^\mu\}$



$: 2iq^2 g_{\mu\nu}$

④ $-\frac{q^2}{2}\{|\phi_+|^4 + |\phi_-|^4 - 2|\phi_+|^2|\phi_-|^2\}$

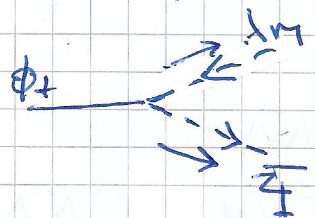
$\rightsquigarrow$ 

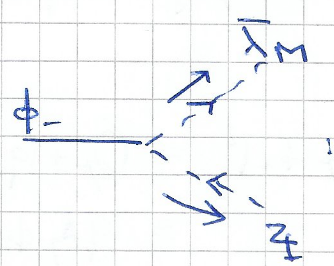
$: -2iq^2$, 

$: iq^2$

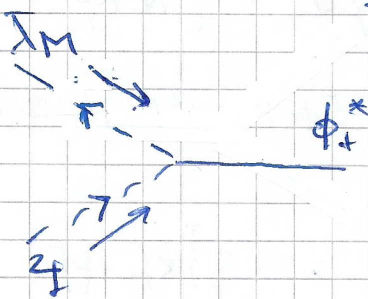
Direction of arrows?

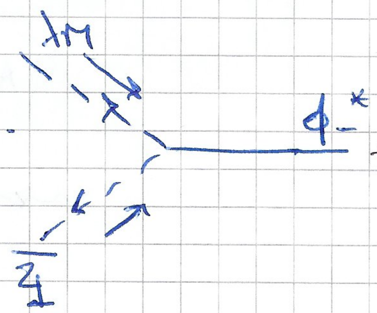
⑤ $-\sqrt{2}q\{\bar{q}P_R\lambda_M\phi_+ - \bar{\lambda}_M P_R q\,\phi_- + \bar{\lambda}_M P_L q\,\phi_+^* - \bar{q}P_L\lambda_M\phi_-^*\}$

$\rightsquigarrow$ 

$: -\sqrt{2}iqP_R$, 

$: \sqrt{2}iqP_R$



$: -\sqrt{2}iqP_L$, 

$: +\sqrt{2}iqP_L$