Disclaimer

The solution at hand was written in the course of the respective class at the University of Bonn. If not stated differently on top of the first page or the following website, the solution was prepared and handed in solely by me, Marvin Zanke. Anything in a different color than the ball pen blue is usually a correction that I or a tutor made. For more information and all my material, check:
https://www.physics-and-stuff.com/

I raise no claim to correctness and completeness of the given solutions! This equally applies to the corrections mentioned above.

Advanced Theoretical Particle Physics Homework 4

12.05.2018

1)
$$\frac{dg_i^2(Q)}{d\ln Q} = \beta_i = -\frac{g_i^4}{8\pi^2} b_i \qquad (*)$$

"chiral fermions"?

$$b_i = \frac{11}{3} C_2(N) - \frac{2}{3} \sum_{\text{ch. ferm}} T(f) - \frac{1}{3} \sum_{\text{Cplx. scalars}} T(S)$$

$= N$ for $SU(N)$ — $\frac{1}{2}$ for fund. rep. of $SU(N)$ — " ___ "

Why $C_2 = 0$ for $U(1)_Y$? No cpl. of $U(1)$ to $U(1)$ g. Bos.? why $T = \frac{1}{2}$ for fund.? $T = Y^2$ for $U(1)$?
⇝ group factor from e.g. λ^a Gellmann yields $T(R)$, which is $\frac{1}{2}$ for fund rep.
Y^2 by looking at the diagr. for $U(1)$, which yields $\sim e^2$ for vertices for QED e.g.
in $U(1)$ $C = 0$ because group factor ⇝ only for non-abelian

While for $U(1)_Y$: $C_2 = 0$ and $T \mathrel{\hat=} Y^2$

$$(*) \Longleftrightarrow \frac{dg_i^2(Q)}{g_i^4} = -\frac{b_i}{8\pi^2} d\ln Q \qquad \Big| \int \text{both sides}, \ (g_i^2)^2 = g_i^4$$

$$\Longleftrightarrow -\frac{1}{g_i^2}\Big|_{M_Z}^{Q} = -\frac{b_i}{8\pi^2} \ln Q \Big|_{M_Z}^{Q}$$

$$\Longleftrightarrow -\frac{1}{g_i^2(Q)} + \frac{1}{g_i^2(M_Z)} = -\frac{b_i}{8\pi^2} \ln\frac{Q}{M_Z}$$

$$\rightsquigarrow \frac{1}{g_i^2(Q)} = \frac{1}{g_i^2(M_Z)} + \frac{b_i}{8\pi^2} \ln\frac{Q}{M_Z}$$

Unification into $SU(5)$ like group? Why does this mean that $g_1 \sim g_Y$? ⇝ in $\mathcal{L}$ only terms $\sim Y g_Y$ appear and we derived how g_1 is normalized by looking at $SU(5)$. g_Y by itself is not well-defined

M_X: unification scale where $SU(2)_L$ and $SU(3)_C$ gauge cpls. meet

M_X': unification scale where g_2 and $g_1 = \sqrt{\frac{5}{3}} g_Y$ meet

First, we calculate $b_{i=1,2,3} / b_Y$, noticing that with $g_1 = \sqrt{\frac{5}{3}} g_Y$, we find $g_Y^2 = \frac{3}{5} g_1^2$ and thus from $(*)$ that $b_1 = \frac{3}{5} b_Y \qquad (**)$
(factor $3/5$ cancels on both sides to get $\frac{dg_1^2(Q)}{d\ln Q} = -\frac{g_1^4}{8\pi^2} b_1$)

For $SU(3)$ e.g. fixed color ⇝ no color factor and $SU(2)$ then fixed what? $SU(2)$ does differ between the color as every color can be in the loop?! Doesn't couple to Higgs for $SU(3)$?
⇝ for $SU(2)$ the dublett is just a number because $SU(2)$ trafo makes it stay a dublett
$SU(2)$ sees dublett as one, $SU(3)$ sees triplet of colors as one

$$b_3 = 11 - \frac{1}{3} \cdot \underbrace{3}_{\text{family}} \cdot \Big\{ \underbrace{1}_{u_L\text{-like}} + \underbrace{1}_{d_L\text{-like}} + \underbrace{1}_{u_R} + \underbrace{1}_{d_R} \Big\} = 7$$

$$b_2 = \frac{22}{3} - \frac{1}{3} \cdot \underbrace{3}_{\text{families}} \Big\{ \underbrace{3}_{\text{color}} \cdot \underbrace{1}_{u_L d_L\text{-dublett}} + \underbrace{1}_{e, \nu_e\text{-dublett}} \Big\} - \frac{1}{6} \Big\{ \underbrace{1}_{\text{Higgs dublett}} \Big\} = \frac{19}{6}$$

$$b_Y = 0 - \frac{2}{3} \cdot \underbrace{3}_{\text{families}} \Big\{ \underbrace{3}_{\text{color}} \cdot \underbrace{2}_{u_L\text{-}d_L\text{ dublett}} \cdot \left(\frac{1}{6}\right)^2 + \underbrace{3}_{\text{Color}} \cdot \underbrace{\left(\frac{2}{3}\right)^2}_{u_R\text{-like}} + \underbrace{3}_{\text{color}} \cdot \underbrace{\left(\frac{1}{3}\right)^2}_{d_R} + 2 \cdot \underbrace{\left(\frac{1}{2}\right)^2}_{e, \nu\text{-dublett}} + \underbrace{(-1)^2}_{e_R} \Big\} - \frac{1}{3} \Big\{ \underbrace{2 \cdot \left(\frac{1}{2}\right)^2}_{\text{Higgs dublett}} \Big\}$$

$$= -\frac{41}{6}$$

$\rightsquigarrow b_1 = -\frac{41}{10}$ using $(**)$

Setting the g_2 and g_1 couplings equal, we find for M_X':

$$\frac{1}{g_1^2(M_Z)} + \frac{b_1}{8\pi^2} \ln \frac{M_X'}{M_Z} = \frac{1}{g_2^2(M_Z)} + \frac{b_2}{8\pi^2} \ln \frac{M_X'}{M_Z}$$

$$\Leftrightarrow \ln \frac{M_X'}{M_Z} \left\{ \frac{b_2 - b_1}{8\pi^2} \right\} = \frac{1}{g_1^2(M_Z)} - \frac{1}{g_2^2(M_Z)}$$

$$\leadsto \ln \frac{M_X'}{M_Z} = \frac{8\pi^2}{b_2 - b_1} \left\{ \frac{1}{g_1^2(M_Z)} - \frac{1}{g_2^2(M_Z)} \right\} = \frac{8\pi^2}{b_2 - b_1} \left\{ \frac{3}{5} \frac{1}{g_Y^2(M_Z)} - \frac{1}{g_2^2(M_Z)} \right\}$$

From the lecture, we take the values $g_3^2(M_Z) = 1{,}50$, $g_2^2(M_Z) = 0{,}421$ and $g_Y^2(M_Z) = 0{,}128 \leadsto g_1^2(M_Z) = \frac{16}{75}$

and find

$$\ln \frac{M_X'}{M_Z} \approx 25{,}124 \quad \text{and with } M_Z = 92 \text{ GeV}$$

$$M_X' \simeq 7{,}495 \cdot 10^{12} \text{ GeV}$$

Why do we know $g_i(M_Z)$ at some spec. scale? $\leadsto$ do exp. (c.o.m. system) at this energy

Still higher than collider energies? $\leadsto$ yes! $\sim 10^3$ GeV max. right now (14 TeV)

When we assume $g_3(M_X') = g_2(M_X')$ (i.e. unification of all 3 cpls.), we look at

$$\frac{1}{g_3^2(M_X')} = \frac{1}{g_3^2(M_Z)} + \frac{b_3}{8\pi^2} \ln \frac{M_X'}{M_Z} \quad \text{and find}$$

$$\leadsto \frac{1}{g_3^2(M_Z)} = \frac{1}{g_2^2(M_X')} - \frac{b_3}{8\pi^2} \ln \frac{M_X'}{M_Z}$$

$$= \frac{1}{g_2^2(M_Z)} + \ln \frac{M_X'}{M_Z} \frac{b_2 - b_3}{8\pi^2} \approx 1{,}156$$

0,865! Inverse!

$$\leadsto \frac{1{,}5 - 1{,}156}{1{,}156} \hat{=} 29\% \text{ off from exp. value}$$

Even worse prediction? $\leadsto$ yes!

b)

Adding (extra light) fields from a complete $\underline{5}$ or $\underline{10}$ of $SU(5)$ obviously changes the β_i-fct., as we get more contributions to the chiral fermion sum. Yet, looking at

$$\frac{1}{g_i^2(Q)} = \frac{1}{g_i^2(M_Z)} + \frac{b_i}{8\pi^2}\ln\frac{Q}{M_Z}$$ and for the scales thus

$$\frac{1}{g_3^2(M_Z)} + \frac{b_3}{8\pi^2}\ln\frac{M_X}{M_Z} = \frac{1}{g_2^2(M_Z)} + \frac{b_2}{8\pi^2}\ln\frac{M_X}{M_Z}$$ and

$$\frac{1}{g_2^2(M_Z)} + \frac{b_2}{8\pi^2}\ln\frac{M_X'}{M_Z} = \frac{1}{g_1^2(M_Z)} + \frac{b_1}{8\pi^2}\ln\frac{M_X'}{M_Z}$$, yielding

$$\ln\frac{M_X}{M_Z} = \frac{8\pi^2}{b_3-b_2}\left\{\frac{1}{g_2^2(M_Z)} - \frac{1}{g_3^2(M_Z)}\right\}$$ and

$$\ln\frac{M_X'}{M_Z} = \frac{8\pi^2}{b_1-b_2}\left\{\frac{1}{g_2^2(M_Z)} - \frac{1}{g_1^2(M_Z)}\right\}$$, we find that if

$b_3 - b_2$ and $b_1 - b_2$ stay the same, the energy scales also remain the same. $(b_1 = \frac{3}{5} b_Y)$

Adding a $\underline{5}$ $(\hat{=} \underbrace{(3,1)}_{d_R \text{ from lecture}} + \underbrace{(1,2)}_{e_L})$

Add 3x for generations, otherwise mess up flavor structure

$$b_3 \longmapsto b_3 - \frac{1}{3}\{1\} = b_3 - \frac{1}{3} \quad \times 3$$

$$b_2 \longmapsto b_2 - \frac{1}{3}\{1\} = b_2 - \frac{1}{3} \quad \times 3$$

$$b_Y \longmapsto b_Y - \frac{2}{3}\left\{3\cdot\left(\frac{1}{3}\right)^2 + 2\left(\frac{1}{2}\right)^2\right\} = b_Y - \frac{5}{9} \quad \times 3$$

$$b_1 \longmapsto b_1 - \frac{1}{3} \implies \Delta b_i\text{'s} = 0$$

$\underline{5} = (3,1)_{-1/3} \oplus (1,2)_{1/2}$

$\underline{5} = (\bar{3},1)_{1/3} \oplus (1,2)_{-1/2}$ (d^c, L)

Adding a $\underline{10}$ $(\hat{=} \underbrace{(3,2)}_{q_L} + \underbrace{(\bar{3},1)}_{u_R^c} + \underbrace{(1,1)}_{e_R^c})$

$$b_3 \longmapsto b_3 - \frac{1}{3}\{2\cdot 1 + 1 + 1\} = b_3 - 1$$

$SU(3)$ triplet, $SU(2)$ doublet

$$b_2 \longmapsto b_2 - \frac{1}{3}\{3\} = b_2 - 1$$

$$b_Y \longmapsto b_Y - \frac{2}{3}\left\{3\cdot 2\cdot\left(\frac{1}{6}\right)^2 + 3\left(\frac{2}{3}\right)^2 + (-1)^2\right\} = b_Y - \frac{5}{3}$$

$$\leadsto b_1 \longmapsto b_1 - 1 \implies \Delta b_i\text{'s} = 0$$

Why extra light fields but $\geq M_Z$? $\leadsto$ for unification scale those are light fields — know cpls. at a lower scale than M_Z. Different reps. from $SU(5)$, $\underline{5}$, $\underline{10}$ w/ more particles in $\underline{10}$? Rationally?

Don't add Higgs-fields? $\leadsto$ changes the breaking; won't break to $SU(3)\times SU(2)\times U(1)$

Add $(\bar{3},1)$ or $(3,1)$ here, i.e. d_R or d_R^c? $\leadsto$ doesn't change anything

Why does $(1,2)$ (e.g. e_L) not couple to $SU(3)$ at all while $(3,1)$ (e.g. u_R) couples $SU(3)$? $\leadsto$ Doesn't

Add also identify $(1,2)$ w/ other particle(s)? $\leadsto$ want SM content

c) considering $\ln \frac{M_X'}{M_Z} = \left(\frac{1}{g_1^2(M_Z)} - \frac{1}{g_2^2(M_Z)} \right) \frac{8\pi^2}{b_2 - b_1}$

$$\ln \frac{M_X}{M_Z} = \left(\frac{1}{g_2^2(M_Z)} - \frac{1}{g_3^2(M_Z)} \right) \frac{8\pi^2}{b_3 - b_2}$$

for $M_X' = M_X$

$$\rightsquigarrow \left(\frac{1}{g_1^2(M_Z)} - \frac{1}{g_2^2(M_Z)} \right) \frac{1}{b_2 - b_1} = \left(\frac{1}{g_2^2(M_Z)} - \frac{1}{g_3^2(M_Z)} \right) \frac{1}{b_3 - b_2}$$

$$\Leftrightarrow (b_3 - b_2) \left(\frac{1}{g_1^2(M_Z)} - \frac{1}{g_2^2(M_Z)} \right) = (b_2 - b_1) \left(\frac{1}{g_2^2(M_Z)} - \frac{1}{g_3^2(M_Z)} \right)$$

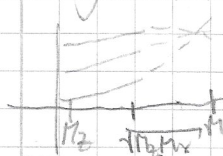
$$\Leftrightarrow b_3 \underbrace{\left(\frac{1}{g_1^2(M_Z)} - \frac{1}{g_2^2(M_Z)} \right)}_{2,312203088} = b_2 \underbrace{\left(\frac{1}{g_1^2(M_Z)} - \frac{1}{g_3^2(M_Z)} \right)}_{\frac{193}{48}} + b_1 \underbrace{\left(\frac{1}{g_3^2(M_Z)} - \frac{1}{g_2^2(M_Z)} \right)}_{-\frac{2158}{1263}}$$

$$\rightsquigarrow b_3 = 1,738962012\, b_2 - 0,7389620117\, b_1$$

Why near M_Z?

$\rightsquigarrow \sqrt{M_Z M_X}$

geom. mean (closer to big value)



the running of cpl. would change: up to $\sqrt{M_X M_Z}$ only SM particles; then new particles have to be included to β-fct; if near M_Z, stays the same

$\rightsquigarrow$ Two-step matching

first match from M_Z to $\sqrt{M_X M_Z}$, then use new β-fct from $\sqrt{M_X M_Z}$ to M_X

$$\frac{1}{g_i^2(M_Z)} - \frac{1}{g_i^2(\sqrt{M_Z M_X})} = b_i \ln\left(\frac{\sqrt{M_X M_Z}}{M_Z} \right)$$

2) a)

$$\mathcal{L}_{10} = -tr\{\underbrace{\overline{\chi}_{10}\, i\not{\partial}\chi_{10}}_{\mathcal{L}_{10,kin}} - \underbrace{2g_5\,\overline{\chi}_{10} V^a_\mu \gamma^\mu t^a \chi_{10}}_{\mathcal{L}_{10,int}}\}$$

10-dim. fermionic rep. χ_{10}? Difference to $\mathcal{L}^5_{Yang}$? → Some of the particles are in 5, some in 10. need diff irreps of SU(5) to get all SM particles in there, see diff quantum no's under SU(3), SU(2), ...

We will use that

$$\overline{q^c_R}\not{\partial} q^c_R = \overline{q^c_R}\partial_\mu\gamma^\mu q^c_R \quad \text{where } q^c = C\overline{q}^T$$

$$= q^{c\dagger}_R\gamma^0\partial_\mu\gamma^\mu q^c_R = \overline{q_R}^* C^\dagger\gamma^0\partial_\mu\gamma^\mu C\overline{q_R}^T$$

$$\overset{()^T}{=} -\overline{q_R}\, C^T\partial_\mu\gamma^{\mu T}\gamma^{0T}C^*\overline{q_R}^\dagger$$

(minus from transposing fermionic fields?)

$$\left|\ \underline{C^{-1} = C^\dagger},\quad C = (C^\dagger)^{-1},\quad C^* = (C^T)^{-1} = -C^{-1} \quad (C^T = -C)\right.$$

$$= \overline{q_R}\, C\gamma^{\mu T}C^{-1}\partial_\mu C\gamma^{0T}C^{-1}\overline{q_R}^\dagger$$

Turned a right handed to left handed $\overline{q^c_R}\not{\partial}q^c_R \mapsto \overline{q_R}\not{\partial}q_R$

$$\left|\ C\Gamma^T C^{-1} = \eta_\Gamma\Gamma,\quad \eta_\Gamma = 1 \text{ if } \Gamma\in\{\mathbb{1}, \gamma_5, \gamma_\mu\gamma_5\},\quad \eta_\Gamma = -1 \text{ if } \Gamma\in\{\gamma_\mu, \sigma_{\mu\nu}\}\right.$$

$$= \overline{q_R}\gamma^\mu\partial_\mu\gamma^0(\gamma^0)^\dagger q_R = \overline{q_R}\not{\partial}q_R$$

Only left-handed in χ_{10}? → yes, and in $\overline{\chi}_{10}$ only right handed there. "bar" makes r.h. ↔ l.h. as well as $()^c$; e.g. (*) $\overline{(u^c_{3L})} \hat{=}$ r.h. $\overline{(u^c_3)}_R$

then $\mathcal{L}_{10,kin} = -tr\{\overline{\chi}_{10}\, i\not{\partial}\chi_{10}\}$, $\chi_{10} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & u^c_3 & -u^c_2 & -u_1 & -d_1 \\ -u^c_3 & 0 & u^c_1 & -u_2 & -d_2 \\ u^c_2 & -u^c_1 & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^c \\ d_1 & d_2 & d_3 & e^c & 0 \end{pmatrix}_L$

(*)

$$\Rightarrow \mathcal{L}_{10,kin} = -\frac{i}{2}tr\left\{\begin{pmatrix} 0 & \overline{u^c_3} & -\overline{u^c_2} & -\overline{u_1} & -\overline{d_1} \\ -\overline{u^c_3} & 0 & \overline{u^c_1} & -\overline{u_2} & -\overline{d_2} \\ \overline{u^c_2} & -\overline{u^c_1} & 0 & -\overline{u_3} & -\overline{d_3} \\ \overline{u_1} & \overline{u_2} & \overline{u_3} & 0 & -\overline{e^c} \\ \overline{d_1} & \overline{d_2} & \overline{d_3} & \overline{e^c} & 0 \end{pmatrix}\cdot\begin{pmatrix} 0 & \not{\partial}u^c_3 & -\not{\partial}u^c_2 & -\not{\partial}u_1 & -\not{\partial}d_1 \\ -\not{\partial}u^c_3 & 0 & \not{\partial}u^c_1 & -\not{\partial}u_2 & -\not{\partial}d_2 \\ \not{\partial}u^c_2 & -\not{\partial}u^c_1 & 0 & -\not{\partial}u_3 & -\not{\partial}d_3 \\ \not{\partial}u_1 & \not{\partial}u_2 & \not{\partial}u_3 & 0 & -\not{\partial}e^c \\ \not{\partial}d_1 & \not{\partial}d_2 & \not{\partial}d_3 & \not{\partial}e^c & 0 \end{pmatrix}\right\}$$

What is u_1, u_2, u_3 for? color? → yes?!

in χ_{10} only l.h. but $()^c$ r.h. → l.h. and $-$ l.h. → r.h.

$$= \frac{i}{2}\Big\{-\overline{u^c_{3,R}}\not{\partial}u^c_{3,R} - \overline{u^c_{2,R}}\not{\partial}u^c_{2,R} - \overline{u_{1,L}}\not{\partial}u_{1,L} - \overline{d_{1,L}}\not{\partial}d_{1,L} - \overline{u^c_{3,R}}\not{\partial}u^c_{3,R}$$
$$- \overline{u^c_{1,R}}\not{\partial}u^c_{1,R} - \overline{u_{2,L}}\not{\partial}u_{2,L} - \overline{d_{2,L}}\not{\partial}d_{2,L} - \overline{u^c_{2,R}}\not{\partial}u^c_{2,R} - \overline{u^c_{1,R}}\not{\partial}u^c_{1,R}$$
$$- \overline{u_{3,L}}\not{\partial}u_{3,L} - \overline{d_{3,L}}\not{\partial}d_{3,L} - \overline{u_{1,L}}\not{\partial}u_{1,L} - \overline{u_{2,L}}\not{\partial}u_{2,L} - \overline{u_{3,L}}\not{\partial}u_{3,L}$$
$$- \overline{e^c_R}\not{\partial}e^c_R - \overline{d_{1,L}}\not{\partial}d_{1,L} - \overline{d_{2,L}}\not{\partial}d_{2,L} - \overline{d_{3,L}}\not{\partial}d_{3,L} - \overline{e^c_R}\not{\partial}e^c_R\Big\}$$

$$\stackrel{\text{hint}}{=} -\frac{i}{2}\Big\{ 2\overline{u_{3,R}}\not\partial u_{3,R} + 2\overline{u_{2,R}}\not\partial u_{2,R} + 2\overline{u_{1,R}}\not\partial u_{1,R}$$
$$+2\overline{u_{3,L}}\not\partial u_{3,L} + 2\overline{u_{2,L}}\not\partial u_{2,L} + 2\overline{u_{1,L}}\not\partial u_{1,L}$$
$$+2\overline{d_{3,L}}\not\partial d_{3,L} + 2\overline{d_{2,L}}\not\partial d_{2,L} + 2\overline{d_{1,L}}\not\partial d_{1,L} + 2\overline{e_R}\not\partial e_R\Big\}$$

(-) sign wrong?

b) $$\mathcal{L}_{10,int,12} \stackrel{a=12}{=} 2g_5\,\mathrm{tr}\{\overline{\chi_{10}}B_\mu\gamma^\mu t^{12}\chi_{10}\}, \quad t^{12} = \frac{1}{2\sqrt{15}}\begin{pmatrix} 2 & & & & 0 \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ 0 & & & & -3 \end{pmatrix}$$

Can combine the u_L, u_R terms but not for d?

$$\Rightarrow \mathcal{L}_{10,int,12} = \frac{-2g_5B_\mu}{4\sqrt{15}}\,\mathrm{tr}\left\{\begin{pmatrix} 0 & \overline{u_3^c} & -\overline{u_2^c} & -\overline{u_1} & -\overline{d_1} \\ -\overline{u_3^c} & 0 & \overline{u_1^c} & -\overline{u_2} & -\overline{d_2} \\ \overline{u_2^c} & -\overline{u_1^c} & 0 & -\overline{u_3} & -\overline{d_3} \\ \overline{u_1} & \overline{u_2} & \overline{u_3} & 0 & -\overline{e^c} \\ \overline{d_1} & \overline{d_2} & \overline{d_3} & \overline{e^c} & 0 \end{pmatrix}\gamma^\mu\begin{pmatrix} 0 & 2u_3^c & -2u_2^c & -2u_1 & -2d_1 \\ -2u_3^c & 0 & 2u_1^c & -2u_2 & -2d_2 \\ 2u_2^c & -2u_1^c & 0 & -2u_3 & -2d_3 \\ -3u_1 & -3u_2 & -3u_3 & 0 & 3e^c \\ -3d_1 & -3d_2 & -3d_3 & -3e^c & 0 \end{pmatrix}\right\}$$

$g_5 = \sqrt{\frac{5}{3}}g_Y$

$$\downarrow\; = -\frac{g_Y}{6}B_\mu\Big\{-2\overline{u^c_{3,R}}\gamma^\mu u^c_{3,R} - 2\overline{u_2^c}\gamma^\mu u_2^c + 3\overline{u_{1,L}}\gamma^\mu u_{1,L} + 3\overline{d_{1,L}}\gamma^\mu d_{1,L}$$
$$-2\overline{u^c_{3,R}}\gamma^\mu u^c_{3,R} - 2\overline{u^c_{1,R}}\gamma^\mu u^c_{1,R} + 3\overline{u_{2,L}}\gamma^\mu u_{2,L} + 3\overline{d_{2,L}}\gamma^\mu d_{2,L}$$
$$-2\overline{u^c_{2,R}}\gamma^\mu u^c_{2,R} - 2\overline{u^c_{1,R}}\gamma^\mu u^c_{1,R} + 3\overline{u_{3,L}}\gamma^\mu u_{3,L} + 3\overline{d_{3,L}}\gamma^\mu d_{3,L}$$
$$-2\overline{u_{1,L}}\gamma^\mu u_{1,L} - 2\overline{u_{2,L}}\gamma^\mu u_{2,L} - 2\overline{u_{3,L}}\gamma^\mu u_{3,L} + 3\overline{e^c_R}\gamma^\mu e^c_R$$
$$-2\overline{d_{1,L}}\gamma^\mu d_{1,L} - 2\overline{d_{2,L}}\gamma^\mu d_{2,L} - 2\overline{d_{3,L}}\gamma^\mu d_{3,L} + 3\overline{e^c}\gamma^\mu e^c\Big\}$$

$$= -\frac{g_Y}{6}B_\mu\Big\{-4\overline{u_{3,R}}\gamma^\mu u_{3,R} - 4\overline{u_{2,R}}\gamma^\mu u_{2,R} - 4\overline{u_{1,R}}\gamma^\mu u_{1,R}$$
$$+\overline{u_{3,L}}\gamma^\mu u_{3,L} + \overline{u_{2,L}}\gamma^\mu u_{2,L} + \overline{u_{1,L}}\gamma^\mu u_{1,L}$$
$$+\overline{d_{3,L}}\gamma^\mu d_{3,L} + \overline{d_{2,L}}\gamma^\mu d_{2,L} + \overline{d_{1,L}}\gamma^\mu d_{1,L} + 6\overline{e_R}\gamma^\mu e_R\Big\} \checkmark$$

$\Rightarrow$ factors $Y(e_R^c) = 1$, $Y(u_R^c) = -\frac{2}{3}$, $Y(q_L) = \frac{1}{6}$

No operator γ here? As $\overline{e_R^c}\gamma^\mu e_R^c = \overline{e_R}\gamma^\mu e_R$, this yields $Y(e_R^c) = Y(e_R)$?!

$\Rightarrow -\frac{g_Y}{6}B_\mu$ ...

$\stackrel{!}{=} g_Y B_\mu Y(u_3$ ...