Disclaimer

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https://www.physics-and-stuff.com/

I raise no claim to correctness and completeness of the given solutions! This equally applies to the corrections mentioned above.

Advanced theoretical Particle Physics Homework 6

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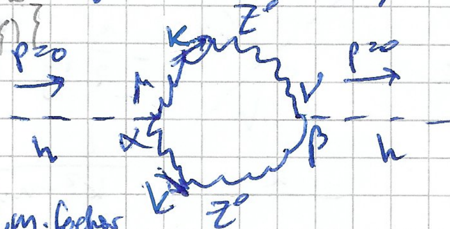
11.06.2018

1) We will use Feynman gauge. The propagator of massive gauge bosons is given by $-ig_{\mu\nu}/(k^2-m_V^2)$, $V=W^\pm, Z^0$

Why Feynman gauge? → convenient in loop calc.; no additional $k_\mu k_\nu$-term $\sim \frac{-i}{k^2-m^2}\{g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2-\xi m^2}(1-\xi)\}$

a) Diagrams w/ gauge boson propagators $\quad \delta m^2 \propto \Lambda^2$, need counter-term to absorb it



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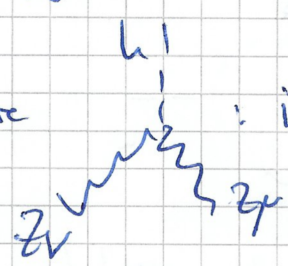


$: i\frac{g}{\cos\theta_W} m_Z g_{\mu\nu}$

Sym. factor

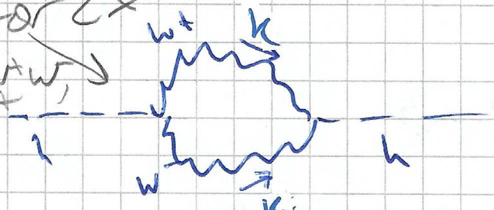
$$= -\frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\left\{i\frac{g}{\cos\theta_W} m_Z g_{\nu\beta}\right\}\frac{g^{\beta\alpha}}{k^2-m_Z^2}\left\{i\frac{g}{\cos\theta_W} m_Z g_{\alpha\mu}\right\}\frac{g^{\mu\nu}}{k^2-m_Z^2}$$

$$= \frac{g^2 m_Z^2}{2\cos^2\theta_W}\int\frac{d^4k}{(2\pi)^4}\frac{g_{\nu\beta}g^{\beta\alpha}g_{\alpha\mu}g^{\mu\nu}}{(k^2-m_Z^2)(k^2-m_Z^2)}$$

$$= \frac{2g^2 m_Z^2}{\cos^2\theta_W}\int\frac{d^4k}{(2\pi)^4}\frac{1}{[k^2-m_Z^2]^2} \sim \log \text{ div.}$$

Why does spin for superparticles differ by 1/2, i.e. why fermions → bosons vice versa? → Poincaré algebra to QFT → get ?

factor 2× for W^+W^-, W^-W^+

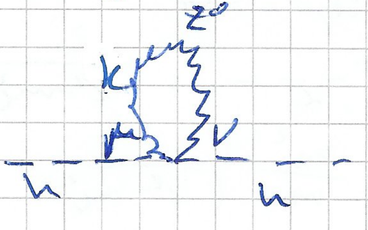


, where

$: i g m_W g_{\mu\nu}$

$$= g^2 m_W^2\int\frac{d^4k}{(2\pi)^4} g_{\nu\beta}\frac{g^{\beta\alpha}}{k^2-m_{W^+}^2} g_{\alpha\mu}\frac{g^{\mu\nu}}{k^2-m_{W^-}^2}$$

$$= 4g^2 m_W^2\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_{W^+}^2}\frac{1}{k^2-m_{W^-}^2} \sim \log \text{ div.}$$



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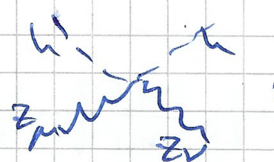


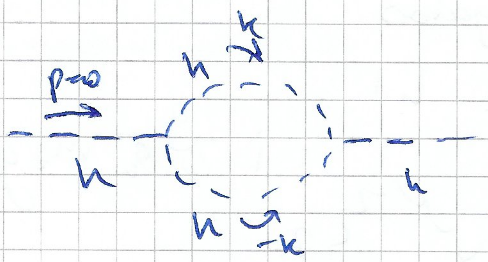
$: \frac{i}{2}\frac{g^2}{\cos^2\theta_W} g_{\mu\nu}$

No Tadpole w/ $W^\pm$? → No contraction like this hhW^+W^+ and thus we also can't assign momenta the other way to make it fit

$$= \frac{1}{2}\int\frac{d^4k}{(2\pi)^4}\left\{\frac{i}{2}\frac{g^2}{\cos^2\theta_W} g_{\mu\nu}\right\}\frac{-ig^{\mu\nu}}{k^2-m_Z^2}$$

$$= \frac{g^2}{\cos^2\theta_W}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_Z^2} \sim \underline{\text{quadr. div.}}$$

b)



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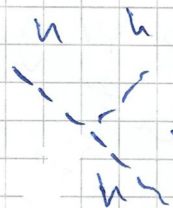
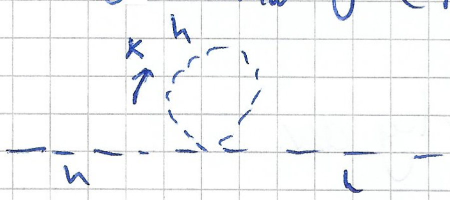


$: -\frac{3}{2} i g \frac{m_h^2}{m_W}$

$: \frac{i}{k^2 - m_h^2}$

$$= \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \left\{ -\frac{3}{2} i g \frac{m_h^2}{m_W} \right\} \frac{i}{k^2 - m_h^2} \left\{ -\frac{3}{2} i g \frac{m_h^2}{m_W} \right\} \frac{i}{k^2 - m_h^2}$$

$$= \frac{9}{8} \frac{g^2 m_h^4}{m_W^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_h^2)^2} \sim \log \text{ div.}$$



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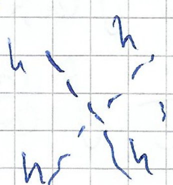
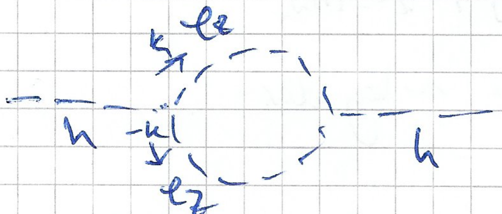


$: -\underbrace{\frac{3}{4} i g^2 \frac{m_h^2}{m_W^2}}_{\frac{i}{4} \frac{M_H^2}{\langle V \rangle^2}}$

$\uparrow$
$g^2 v^2 = m_W^2$

$$= \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \left\{ -\frac{3}{4} i g^2 \frac{m_h^2}{m_W^2} \right\} \frac{i}{k^2 - m_h^2}$$

$$= \frac{3}{8} g^2 \frac{m_h^2}{m_W^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_h^2} \sim \underline{\text{quadr. div}}$$



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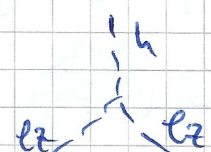


$: -\frac{i}{2} g \frac{m_h^2}{m_W}$

$: \frac{i}{k^2 - m_{\phi}^2}$

$$= \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \left\{ -\frac{i}{2} g \frac{m_h^2}{m_W} \right\} \frac{i}{k^2 - m_{\phi_Z}^2} \left\{ -\frac{i}{2} g \frac{m_h^2}{m_W} \right\} \frac{i}{k^2 - m_{\phi_Z}^2}$$

$$= \frac{g^2}{8} \frac{m_h^4}{m_W^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_{\phi_Z}^2)^2} \sim \log \text{ div.}$$

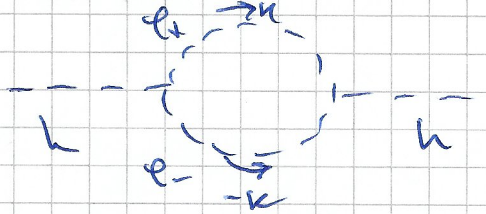
✓ Why can we gauge away the goldstone bosons for local symmetries ⇝ local and thus depends on higgs

✓ Same g as for coupling w/ $W^\pm Z$? ⇝ should be a different cplg. but they expressed all with the same cplg. g

✓ Can we use that $m_\phi = 0$? (As we are not in unitary gauge) ⇝ there is no m_ϕ in the prop as $\Phi = (\phi^+, v + h + i\phi^0)$

$\frac{i}{k^2 - m_W^2}$ $\phi^\pm$

$\frac{i}{k^2 - m_Z^2}$ ϕ^0



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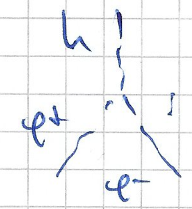
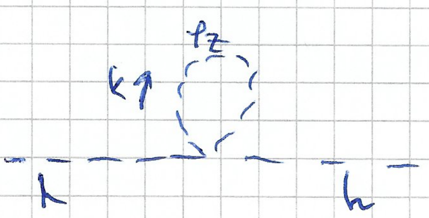


: $-\frac{i}{2} g \frac{m_h^2}{m_W}$

$$= \frac{g^2}{4} \frac{m_h^4}{m_W^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_{\varphi^+}^2} \frac{1}{k^2 - m_{\varphi^-}^2} \sim \text{log div.}$$



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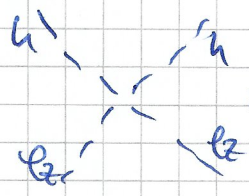
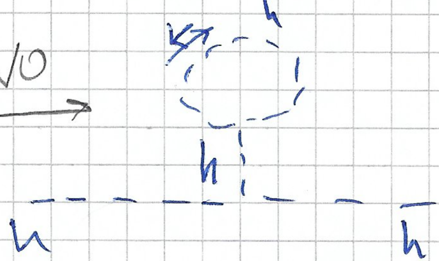


: $-\frac{i}{4} g^2 \frac{m_h^2}{m_W^2}$

$$= \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \left\{ -\frac{i}{4} g^2 \frac{m_h^2}{m_W^2} \right\} \frac{i}{k^2 - m_{\varphi_z}^2}$$

$$= \frac{g^2}{8} \frac{m_h^2}{m_W^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_{\varphi_z}^2} \sim \underline{\text{quadr. div.}}$$

NO →



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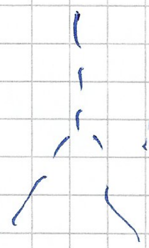


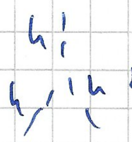
: $-\frac{3}{2} i g \frac{m_h^2}{m_W}$

$$= \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \left\{ -\frac{3}{2} i g \frac{m_h^2}{m_W} \right\} \left\{ \frac{i}{-m_h^2} \right\} \left\{ -\frac{3}{2} i g \frac{m_h^2}{m_W} \right\} \frac{i}{k^2 - m_h^2}$$

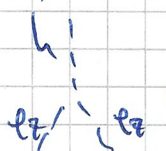
$$= -\frac{9}{8} g^2 \frac{m_h^2}{m_W^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_h^2} \sim \underline{\text{quadr. div.}}$$

No →

, where



: $-\frac{3}{2} i g \frac{m_h^2}{m_W}$

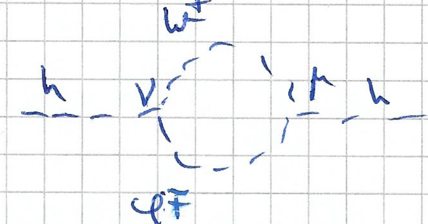


: $-\frac{i}{2} g \frac{m_h^2}{m_W}$

$$= \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \left\{ -\frac{3}{2} i g \frac{m_h^2}{m_W} \right\} \left\{ \frac{i}{-m_h^2} \right\} \left\{ -\frac{i}{2} g \frac{m_h^2}{m_W} \right\} \frac{i}{k^2 - m_{\varphi_z}^2}$$

$$= -\frac{3}{8} g^2 \frac{m_h^2}{m_W^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_{\varphi_z}^2} \sim \underline{\text{quadr. div}}$$

c)



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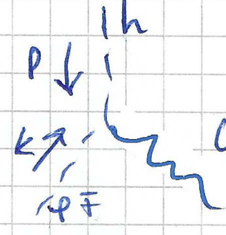
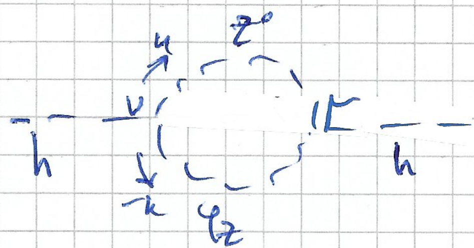


$W^\pm_\mu : \pm\frac{i}{2}\eta g(k-p)_\mu$

Same sign (±) on both vertices? ✓
yes

$$= \int \frac{d^4k}{(2\pi)^4} \left\{ \pm \frac{i}{2}\eta g (\overset{+k}{-k})_\mu \right\} \left\{ \frac{-ig_{\mu\nu}}{k^2-m_{W^\pm}^2}\right\} \left\{\pm\frac{i}{2}\eta g\, k_\nu\right\} \frac{i}{k^2-m_{\varphi^\mp}^2}$$

$$= \frac{\eta^2 g^2}{4}\int \frac{d^4k}{(2\pi)^4}\frac{k^2}{(k^2-m_{W^\pm}^2)(k^2-m_{\varphi^\mp}^2)} \sim \underline{\text{quadr. div}}$$



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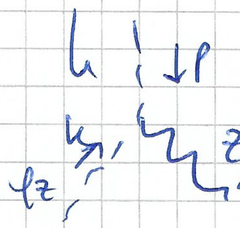
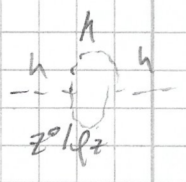


$: -\eta\eta_Z\frac{g}{2\cos\theta_W}(k-p)_\mu$

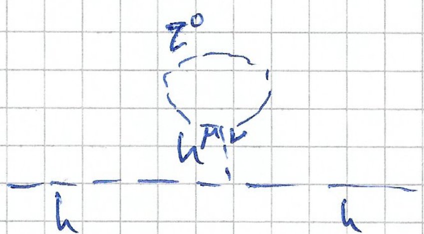
"previous sub-section"?

$$= \int \frac{d^4k}{(2\pi)^4}\left\{-\eta\eta_Z \frac{g}{2\cos\theta_W}(-k)_\mu\right\}\left\{\frac{-ig_{\mu\nu}}{k^2-m_Z^2}\right\}\left\{-\eta\eta_Z\frac{g}{2\cos\theta_W}k_\nu\right\}\frac{i}{k^2-m_{\varphi_Z}^2}$$

No ?



$$= -\eta^2\eta_Z^2 \frac{g^2}{4\cos^2\theta_W}\int \frac{d^4k}{(2\pi)^4}\frac{k^2}{(k^2-m_Z^2)(k^2-m_{\varphi_Z}^2)} \sim \underline{\text{quadr. div.}}$$



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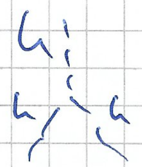
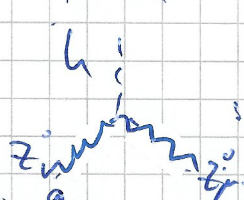


$: -\frac{3}{2}ig\frac{m_h^2}{m_W}$



$: i\frac{g}{\cos\theta_W}m_Z g_{\mu\nu}$

$$= \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\left\{-\frac{3}{2}ig\frac{m_h^2}{m_W}\right\}\left\{\frac{i}{-m_h^2}\right\}\left\{i\frac{g}{\cos\theta_W}m_Z g_{\mu\nu}\right\}\frac{ig^{\mu\nu}}{k^2-m_Z^2}$$

$$= \frac{12}{4}\frac{g^2}{\cos\theta_W}\frac{m_Z}{m_W}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_Z^2} \sim \underline{\text{quadr. div.}}$$

d) (1) in a) $\frac{g^2}{\cos^2\theta_W}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_Z^2}$

(2) in b) $\frac{3g^2}{8}\frac{m_h^2}{m_W^2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_h^2}$

(3) in b) $\frac{g^2}{8}\frac{m_h^2}{m_W^2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-\cancel{m_{\xi_Z}^2}}$

(4) in b) $-\frac{9g^2}{8}\frac{m_h^2}{m_W^2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_h^2}$

(5) in b) $-\frac{3g^2}{8}\frac{m_h^2}{m_W^2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-\cancel{m_{\xi_Z}^2}}$

$\eta^2=1$ (6) in c) $\frac{g^2}{4}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_{W^\pm}^2}$

$\eta_Z^2=1$ (7) in c) $\frac{g^2}{4\cos^2\theta_W}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_Z^2}$

(8) in c) $\frac{3g^2}{\cos\theta_W}\frac{m_Z}{m_W}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_Z^2}$

where from Wolfram Alpha,

What if (k^2-c^2) in $(0,\Lambda)$?

$$\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-c^2}\;\hat{=}\;\int\frac{dk}{(2\pi)^4}\frac{k^3}{k^2-c^2}=\frac{1}{2}\left\{c^2\log(k^2-c^2)+k^2\right\}\Big|_0^\Lambda$$

$\rightsquigarrow$ quadr. divergence $\sim\Lambda^2$ for all integrals (indep. of c^2)