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Advanced theoretical Particle Physics Homework 9

20.06.2018 1) Frame in which a massive particle at rest, $P_\mu = (m,0,0,0)$

Why one particle? Where is the super-multiplet – or why have one particle of a supermultiplet? ⇝ Supermultiplet by applying Q

a) We consider the supercharges $Q_1, Q_2, \bar{Q}_{\dot 1}, \bar{Q}_{\dot 2}$

From the lecture, we know:

$$\{Q_A, \bar{Q}_{\dot B}\} = 2\sigma^\mu_{A\dot B} P_\mu \xrightarrow{\text{lecture}} \{\bar{Q}^{\dot A}, Q^B\} = 2\bar{\sigma}^{\mu \dot A B} P_\mu$$

$$\{Q_A, Q_B\} = \{\bar{Q}^{\dot A}, \bar{Q}^{\dot B}\} = 0$$

$Q = \begin{pmatrix} Q_1 \\ Q_2 \\ \bar{Q}_{\dot 1} \\ \bar{Q}_{\dot 2} \end{pmatrix}$? ⇝ $Q_s = \begin{pmatrix} Q_1 \\ Q_2 \\ \bar{Q}_{\dot 1} \\ \bar{Q}_{\dot 2} \end{pmatrix}$

why in lecture indices for lower comp. up but here (and in lecture's result) down? ⇝ pull indices w/ ε

$$\rightsquigarrow \{Q_A, \bar{Q}_{\dot B}\} = 2\sigma^0_{A\dot B} m = 2m\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_{A\dot B}$$

+1

$$\rightsquigarrow \{Q_1, \bar{Q}_{\dot 1}\} = 2m, \quad \{Q_2, \bar{Q}_{\dot 2}\} = -2m$$

+2m

$$\{Q_1, \bar{Q}_{\dot 2}\} = 0 = \{Q_2, \bar{Q}_{\dot 1}\}$$

$$\{Q_1, Q_1\} = 0 = \{Q_2, Q_2\}$$

$$\{Q_1, Q_2\} = 0 = \{Q_2, Q_1\}$$

How to get $[J^3, \bar{Q}^{\dot A}]$? Equal to $[J^3, Q_A]$?

$$\{\bar{Q}_{\dot 1}, \bar{Q}_{\dot 1}\} = 0 = \{\bar{Q}_{\dot 2}, \bar{Q}_{\dot 2}\}$$ ← indices up?

$$\{\bar{Q}_{\dot 1}, \bar{Q}_{\dot 2}\} = 0 = \{\bar{Q}_{\dot 2}, \bar{Q}_{\dot 1}\}$$

how to get from first to second anticommutator? ⇝ raise indices ⇝ $\sigma \mapsto \bar{\sigma}$ (see appendix)

b) From the lecture, we know that

$$[J^p, Q_A] = -\frac{1}{2}(\sigma^p)_A{}^B Q_B$$

$$[J^p, \bar{Q}^{\dot A}] = -\frac{1}{2}(\bar{\sigma}^p)^{\dot A}{}_{\dot B} \bar{Q}^{\dot B}$$

$$\rightsquigarrow J^3 \bar{Q}^{\dot 1} = [J^3, \bar{Q}^{\dot 1}] + \bar{Q}^{\dot 1} J^3$$

$$= -\frac{1}{2}(\bar{\sigma}^3)^{\dot 1}{}_{\dot B}\bar{Q}^{\dot B} + \bar{Q}^{\dot 1} J^3, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \frac{1}{2}\bar{Q}^{\dot 1} + \bar{Q}^{\dot 1} J^3 = \bar{Q}^{\dot 1}(J^3 + 1/2)$$

$$J^3 \bar{Q}^{\dot 2} = [J^3, \bar{Q}^{\dot 2}] + \bar{Q}^{\dot 2} J^3$$

$$= -\frac{1}{2}(\bar{\sigma}^3)^{\dot 2}{}_{\dot B}\bar{Q}^{\dot B} + \bar{Q}^{\dot 2} J^3$$

$$= -\frac{1}{2}\bar{Q}^{\dot 2} + \bar{Q}^{\dot 2} J^3 = \bar{Q}^{\dot 2}(J^3 - 1/2)$$

c) We assume the state $|m,0,0\rangle$ not being annihilated by $\bar{Q}^{\dot{1}}$ and $\bar{Q}^{\dot{2}}$. Then: $\bar{Q}^{\dot{1}}|m,0,0\rangle$, $\bar{Q}^{\dot{2}}|m,0,0\rangle$, $\bar{Q}^{\dot{1}}\bar{Q}^{\dot{2}}|m,0,0\rangle$ and $\bar{Q}^{\dot{2}}\bar{Q}^{\dot{1}}|m,0,0\rangle$ are states of the repr.

Now indices up?

Using $\{\bar{Q}^{\dot{A}},\bar{Q}^{\dot{B}}\}=0$, one instantly finds that $\bar{Q}^{\dot{2}}\bar{Q}^{\dot{1}}|m,0,0\rangle = -\bar{Q}^{\dot{1}}\bar{Q}^{\dot{2}}|m,0,0\rangle$ is not a new state, while the others form a quartet repr. as also $\bar{Q}^{\dot{A}}\bar{Q}^{\dot{A}}=0$ from the anticommutator.

$\mathbb{1}|m,0,0\rangle \neq |m,0,0\rangle = \bar{Q}^{\dot{1}}\bar{Q}^{\dot{2}}|m,0,0\rangle$

Parity difference is opp. parity

From the lecture, we also have: $[\bar{Q}^{\dot{A}},P_\mu]=0=-[P_\mu,\bar{Q}^{\dot{A}}]$

$\Rightarrow [P^2,\bar{Q}^{\dot{A}}]=0$

and $[J^\rho,\bar{Q}^{\dot{A}}] = -\frac{1}{2}(\bar{\sigma}^\rho)^{\dot{A}}{}_{\dot{B}}\bar{Q}^{\dot{B}}$ from before (lecture)

Starting from $|m,0,0\rangle$, we have:

$$P^2|m,0,0\rangle = m^2|m,0,0\rangle$$
$$J^3|m,0,0\rangle = 0|m,0,0\rangle$$
$$J^2|m,0,0\rangle = 0(0+1)|m,0,0\rangle$$

$\Rightarrow$ $\underline{P^2}$: $P^2\bar{Q}^{\dot{1}}|m,0,0\rangle = m^2\bar{Q}^{\dot{1}}|m,0,0\rangle$, $P^2\bar{Q}^{\dot{2}}|m,0,0\rangle = m^2\bar{Q}^{\dot{2}}|m,0,0\rangle$

$P^2\bar{Q}^{\dot{1}}\bar{Q}^{\dot{2}}|m,0,0\rangle = m^2\bar{Q}^{\dot{1}}\bar{Q}^{\dot{2}}|m,0,0\rangle$

$\underline{J^3}$: $J^3\bar{Q}^{\dot{1}}|m,0,0\rangle \overset{(*)}{=} \frac{1}{2}\bar{Q}^{\dot{1}}|m,0,0\rangle$, $J^3\bar{Q}^{\dot{2}}|m,0,0\rangle = -\frac{1}{2}\bar{Q}^{\dot{2}}|m,0,0\rangle$

$J^3\bar{Q}^{\dot{1}}\bar{Q}^{\dot{2}}|m,0,0\rangle = \bar{Q}^{\dot{1}}(J^3+\frac{1}{2})\bar{Q}^{\dot{2}}|m,0,0\rangle$

$= \bar{Q}^{\dot{1}}\bar{Q}^{\dot{2}}J^3|m,0,0\rangle = 0\cdot\bar{Q}^{\dot{1}}\bar{Q}^{\dot{2}}|m,0,0\rangle$

As $Q|m,0,0\rangle$ has $J^3=\frac{1}{2}$

$\Rightarrow J^3_{max}=\frac{1}{2}=j$?

2) Most general superfield taylor expanded in Grassmann coord

$$F(x,\theta,\bar\theta) = f(x) + \sqrt{2}\,\theta\xi(x) + \sqrt{2}\,\bar\theta\bar\chi(x) + \theta\theta M(x) + \bar\theta\bar\theta N(x) + \theta\sigma^\mu\bar\theta A_\mu(x) + \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\zeta(x) + \tfrac{1}{2}\theta\theta\bar\theta\bar\theta D(x)$$

Supersymmetry transformation (shift in superspace)

Why σ^μ in $\tilde{x}$ and thus already contracted with $\partial_\mu\bar\chi(x)$? No free index? $\Rightarrow$ $(x^\mu, \underbrace{\theta,\bar\theta}_{Q})$, x^μ: Lorentz. Want a sup. alg. scalar, i.e. need to contract $\theta\sigma^\mu\bar\epsilon$

$$x \longmapsto x - i\theta\sigma^\mu\bar\epsilon + i\epsilon\sigma^\mu\bar\theta \equiv \tilde{x}$$

$$\theta \longmapsto \theta + \epsilon \equiv \tilde\theta$$

$$\bar\theta \longmapsto \bar\theta + \bar\epsilon \equiv \tilde{\bar\theta}$$

Taylor: $f'(x_0)\{x - x_0\} \Rightarrow$ no x-term

then: $\delta F = F(\tilde{x}, \tilde\theta, \tilde{\bar\theta}) - F(x,\theta,\bar\theta)$ order $\theta, \bar\theta, \epsilon, \bar\epsilon$ linear

Why e.g. $\theta^3 = 0$ if not next to each other? $\theta\theta\chi\theta = \theta^A\theta_A\chi^B\theta_B$ $= \epsilon^{AC}\epsilon^{BD}\theta_C\theta_A\chi_D\theta_B$ $\Rightarrow$ How to com. $\chi_D\theta_B$?

$$\delta F = \partial_\mu f(x)\{\underbrace{-i\theta\sigma^\mu\bar\epsilon}_{(1,0)} + \underbrace{i\epsilon\sigma^\mu\bar\theta}_{(0,1)}\}$$

$$+\underbrace{\sqrt{2}\,\epsilon\xi(x)}_{(0,0)} + \sqrt{2}\,\theta\partial_\mu\xi(x)\{\underbrace{-i\theta\sigma^\mu\bar\epsilon}_{(2,0)} + \underbrace{i\epsilon\sigma^\mu\bar\theta}_{(1,1)}\}$$

$$+\underbrace{\sqrt{2}\,\bar\epsilon\bar\chi(x)}_{(0,0)} + \sqrt{2}\,\bar\theta\partial_\mu\bar\chi(x)\{\underbrace{-i\theta\sigma^\mu\bar\epsilon}_{(1,1)} + \underbrace{i\epsilon\sigma^\mu\bar\theta}_{(0,2)}\}$$

$$+\underbrace{\epsilon\theta M(x)}_{(1,0)} + \underbrace{\theta\epsilon M(x)}_{(1,0)} + \theta\theta\partial_\mu M(x)\{\cancel{-i\theta\sigma^\mu\bar\epsilon} + \underbrace{i\epsilon\sigma^\mu\bar\theta}_{(2,1)}\}$$

$$+\underbrace{\bar\epsilon\bar\theta N(x)}_{(0,1)} + \underbrace{\bar\theta\bar\epsilon N(x)}_{(0,1)} + \bar\theta\bar\theta\partial_\mu N(x)\{\underbrace{-i\theta\sigma^\mu\bar\epsilon}_{(1,2)} + \cancel{i\epsilon\sigma^\mu\bar\theta}\}$$

$$+\underbrace{\epsilon\sigma^\mu\bar\theta A_\mu(x)}_{(0,1)} + \underbrace{\theta\sigma^\mu\bar\epsilon A_\mu(x)}_{(1,0)} + \theta\sigma^\mu\bar\theta\partial_\nu A_\mu(x)\{\underbrace{-i\theta\sigma^\nu\bar\epsilon}_{(2,1)} + \underbrace{i\epsilon\sigma^\nu\bar\theta}_{(1,2)}\}$$

$$+\underbrace{\epsilon\theta\bar\theta\bar\lambda(x)}_{(1,1)} + \underbrace{\theta\epsilon\bar\theta\bar\lambda}_{(1,1)} + \underbrace{\theta\theta\bar\epsilon\bar\lambda(x)}_{(2,0)} + \theta\theta\bar\theta\partial_\mu\bar\lambda(x)\cancel{\{-i\theta\sigma^\mu\bar\epsilon + i\epsilon\sigma^\mu\bar\theta\}}_{(2,2)}$$

$$+\underbrace{\bar\epsilon\bar\theta\theta\zeta(x)}_{(1,1)} + \underbrace{\bar\theta\bar\epsilon\theta\zeta(x)}_{(1,1)} + \underbrace{\bar\theta\bar\theta\epsilon\zeta(x)}_{(0,2)} + \bar\theta\bar\theta\theta\partial_\mu\zeta(x)\{\underbrace{-i\theta\sigma^\mu\bar\epsilon}_{(2,2)} + \cancel{i\epsilon\sigma^\mu\bar\theta}\}$$

$$+\underbrace{\tfrac{1}{2}\epsilon\theta\bar\theta\bar\theta D(x)}_{(1,2)} + \underbrace{\tfrac{1}{2}\theta\epsilon\bar\theta\bar\theta D(x)}_{(1,2)} + \underbrace{\tfrac{1}{2}\theta\theta\bar\epsilon\bar\theta D(x)}_{(2,1)} + \underbrace{\tfrac{1}{2}\theta\theta\bar\theta\bar\epsilon D(x)}_{(2,1)}$$

$$+\theta\theta\bar\theta\bar\theta\partial_\mu D(x)\cancel{\{-i\theta\sigma^\mu\bar\epsilon + i\epsilon\sigma^\mu\bar\theta\}}$$

where some terms cancel because $\theta^3 = \bar\theta^3 = 0$ and $(\#\theta, \#\bar\theta)$ denotes the no. of θ's, $\bar\theta$'s.

We then search of the terms of same order compared to eq (2).

$$\delta(f(x)) = \sqrt{2}\,\epsilon\xi(x) + \sqrt{2}\,\bar{\epsilon}\bar{\chi}(x)$$

$$\delta(\sqrt{2}\,\theta\xi(x)) = -i\partial_\mu f(x)\,\theta\sigma^\mu\bar{\epsilon} + \epsilon\theta M(x) + \theta\epsilon M(x) + \theta\sigma^\mu\bar{\epsilon} A_\mu(x)$$

$$\Big|\ \epsilon\theta = \theta\epsilon \quad (\mathrm{I}6)$$

$$= \theta\sigma^\mu\bar{\epsilon}\{-i\partial_\mu f(x) + A_\mu(x)\} + 2\theta\epsilon M(x)$$

$\theta\xi = \theta^A\xi_A$ (I6)

$$\Rightarrow \delta(\xi_A(x)) = \frac{1}{\sqrt{2}}(\sigma^\mu\bar{\epsilon})_A\{-i\partial_\mu f(x) + A_\mu(x)\} + \sqrt{2}\,\epsilon_A M(x)$$

$$\delta(\sqrt{2}\,\bar{\theta}\bar{\chi}(x)) = i\partial_\mu f(x)\,\epsilon\sigma^\mu\bar{\theta} + \bar{\epsilon}\bar{\theta}N(x) + \bar{\theta}\bar{\epsilon}N(x) + \epsilon\sigma^\mu\bar{\theta}A_\mu(x)$$

$$\Big|\ \epsilon\sigma^\mu\bar{\theta} = -\bar{\theta}\bar{\sigma}^\mu\epsilon \ (\mathrm{I}7a), \quad \bar{\epsilon}\bar{\theta} = \bar{\theta}\bar{\epsilon} \ (\mathrm{I}6)$$

$$= -i\partial_\mu f(x)\,\bar{\theta}\bar{\sigma}^\mu\epsilon + 2\bar{\theta}\bar{\epsilon}N(x) - \bar{\theta}\bar{\sigma}^\mu\epsilon A_\mu(x)$$

$$= -\bar{\theta}\bar{\sigma}^\mu\epsilon\{i\partial_\mu f(x) + A_\mu(x)\} + 2\bar{\theta}\bar{\epsilon}N(x)$$

$\bar{\theta}\bar{\chi} = \bar{\theta}_{\dot{A}}\bar{\chi}^{\dot{A}}$ (I6)

$$\Rightarrow \delta(\bar{\chi}^{\dot{A}}(x)) = -\frac{1}{\sqrt{2}}(\bar{\sigma}^\mu\epsilon)^{\dot{A}}\{i\partial_\mu f(x) + A_\mu(x)\} + \sqrt{2}\,\bar{\epsilon}^{\dot{A}}N(x)$$

$$\delta(\theta\theta M(x)) = -\sqrt{2}\,i\,\theta\partial_\mu\xi(x)\,\theta\sigma^\mu\bar{\epsilon} + \theta\theta\bar{\epsilon}\bar{\lambda}(x)$$

$$\Big|\ \theta\partial_\mu\xi(x)\,\theta\sigma^\mu\bar{\epsilon} = -\tfrac{1}{2}\theta\theta\,\partial_\mu\xi(x)\sigma^\mu\bar{\epsilon} + \underbrace{\theta\sigma^{\mu\nu}\theta}_{=0}\,\partial_\mu\xi(x)\sigma_\nu\bar{\epsilon} \ (\mathrm{I}7e)$$

$$\theta\sigma^{\mu\nu}\theta = -\theta\sigma^{\mu\nu}\theta = 0 \quad (\mathrm{I}7b)$$

$$= -\sqrt{2}\,i\left(-\tfrac{1}{2}\theta\theta\,\partial_\mu\xi(x)\sigma^\mu\bar{\epsilon}\right) + \theta\theta\bar{\epsilon}\bar{\lambda}(x)$$

$$= \frac{i}{\sqrt{2}}\theta\theta\,\partial_\mu\xi(x)\sigma^\mu\bar{\epsilon} + \theta\theta\bar{\epsilon}\bar{\lambda}(x)$$

$$\Rightarrow \delta(M(x)) = \frac{i}{2}\partial_\mu\xi(x)\sigma^\mu\bar{\epsilon} + \bar{\epsilon}\bar{\lambda}(x)$$

$$\delta(\bar\theta\bar\theta N(x)) = \sqrt{2}\, i\, \bar\theta \partial_\mu \bar\chi(x) \overbrace{\epsilon\sigma^\mu\bar\theta}^{-\bar\theta\bar\sigma^\mu\epsilon} + \bar\theta\bar\theta\epsilon\xi(x)$$

$$\bar\zeta\bar\xi\bar\chi\bar\sigma^\mu\tau = -\frac{1}{2}\bar\zeta\bar\chi\bar\xi\bar\sigma^\mu\tau + \bar\zeta\bar\sigma^{\mu\nu}\bar\chi\bar\xi\bar\sigma_\nu\tau \quad \text{(I7e)}$$

but barr'd (daggered) everything.

How to do this daggering formally? Change of order? $\sigma^\dagger \neq \bar\sigma$?

$$= -\sqrt{2}\, i\left(-\frac{1}{2}\bar\theta\bar\theta\partial_\mu\bar\chi(x)\bar\sigma^\mu\epsilon + \bar\theta\bar\sigma^{\mu\nu}\bar\theta\partial_\mu\bar\chi(x)\bar\sigma_\nu\epsilon\right) + \bar\theta\bar\theta\epsilon\xi(x)$$

$$\bar\xi\bar\sigma^{\mu\nu}\bar\chi = -\bar\chi\bar\sigma^{\mu\nu}\bar\xi \rightsquigarrow \bar\theta\bar\sigma^{\mu\nu}\bar\theta = 0 \quad \text{(I7b)}$$

$$= \frac{i}{\sqrt{2}}\bar\theta\bar\theta\underbrace{\partial_\mu\bar\chi(x)\bar\sigma^\mu\epsilon}_{-\epsilon\sigma^\mu\partial_\mu\bar\chi(x)} + \bar\theta\bar\theta\epsilon\xi(x)$$

$$\rightsquigarrow \delta(N(x)) = -\frac{i}{\sqrt{2}}\epsilon\sigma^\mu\partial_\mu\bar\chi(x) + \epsilon\xi(x)$$

$$\delta(\theta\sigma^\mu\bar\theta A_\mu(x)) = \sqrt{2}\, i\,\theta\partial_\mu\xi(x)\epsilon\sigma^\mu\bar\theta - \sqrt{2}\, i\,\bar\theta\partial_\mu\bar\chi(x)\theta\sigma^\mu\bar\epsilon$$
$$+ \bar\epsilon\bar\theta\theta\xi(x) + \bar\theta\bar\epsilon\theta\xi(x) + \epsilon\theta\bar\theta\bar\lambda(x) + \theta\epsilon\bar\theta\bar\lambda(x)$$

$$\bar\epsilon\bar\theta = \bar\theta\bar\epsilon,\ \epsilon\theta = \theta\epsilon \quad \text{(I6)}$$
$$\xi\zeta\bar\chi\bar\tau = \frac{1}{2}\xi\sigma^\mu\bar\chi\,\zeta\sigma_\mu\bar\tau \quad \text{(I7i)}$$
$$\bar\xi\bar\zeta\chi\tau = \frac{1}{2}\bar\xi\bar\sigma^\mu\chi\,\bar\zeta\bar\sigma_\mu\tau \quad \text{(I7j)}$$

$$= \sqrt{2}\, i\,\partial_\mu\xi(x)\theta\epsilon\sigma^\mu\bar\theta + \sqrt{2}\, i\,\partial_\mu\bar\chi(x)\bar\theta\bar\epsilon\bar\sigma^\mu\theta$$
$$+ \bar\theta\bar\sigma^\mu\theta\bar\epsilon\bar\sigma_\mu\xi(x) + \theta\sigma^\mu\bar\theta\epsilon\sigma_\mu\bar\lambda(x)$$

$$\zeta\xi\chi\sigma^\mu\bar\tau = -\frac{1}{2}\zeta\chi\xi\sigma^\mu\bar\tau + \zeta\sigma^{\mu\nu}\chi\xi\sigma_\nu\bar\tau \quad \text{(I7e)}$$
$$\bar\zeta\bar\xi\bar\chi\bar\sigma^\mu\tau = -\frac{1}{2}\bar\zeta\bar\chi\bar\xi\bar\sigma^\mu\tau + \bar\zeta\bar\sigma^{\mu\nu}\bar\chi\bar\xi\bar\sigma_\nu\tau$$

$$= \sqrt{2}\, i\left(-\frac{1}{2}\partial_\mu\xi(x)\epsilon\theta\sigma^\mu\bar\theta + \partial_\mu\xi(x)\sigma^{\mu\nu}\epsilon\theta\sigma_\nu\bar\theta\right)$$
$$+ \sqrt{2}\, i\left(-\frac{1}{2}\partial_\mu\bar\chi(x)\bar\epsilon\bar\theta\bar\sigma^\mu\theta + \partial_\mu\bar\chi(x)\bar\sigma^{\mu\nu}\bar\epsilon\bar\theta\bar\sigma_\nu\theta\right)$$
$$+ \theta\sigma^\mu\bar\theta\xi(x)\sigma_\mu\bar\epsilon + \theta\sigma^\mu\bar\theta\epsilon\sigma_\mu\bar\lambda(x)$$

Why for barr'd term first $\partial_\mu\bar\chi$ while for other ϵ? Can interchange?

$$= \theta\sigma^\mu\bar\theta\left\{-\frac{i}{\sqrt{2}}\epsilon\partial_\mu\xi(x) + \frac{i}{\sqrt{2}}\partial_\mu\bar\chi(x)\bar\epsilon + \xi(x)\sigma_\mu\bar\epsilon + \epsilon\sigma_\mu\bar\lambda(x)\right\}$$
$$+ \sqrt{2}\, i\,\theta\sigma^\mu\bar\theta\left\{\partial^\nu\xi(x)\sigma_{\nu\mu}\epsilon - \partial^\nu\bar\chi(x)\bar\sigma_{\nu\mu}\bar\epsilon\right\}$$

$$\left|\ \sigma^{\nu\mu} = -\sigma^{\mu\nu} \ (I3), \quad \xi\sigma^{\mu\nu}\chi = -\chi\sigma^{\mu\nu}\xi \ (I7b) \right.$$
$$\left|\ \bar{\xi}\bar{\sigma}^{\mu\nu}\bar{\chi} = -\bar{\chi}\bar{\sigma}^{\mu\nu}\bar{\xi} \ (I7b) \right.$$

$$= \theta\sigma^\mu\bar{\theta}\Big\{ -\frac{i}{\sqrt{2}}\epsilon\partial_\mu\xi(x) + \frac{i}{\sqrt{2}}\partial_\mu\bar{\chi}(x)\bar{\epsilon} + \xi(x)\sigma_\mu\bar{\epsilon} + \epsilon\sigma_\mu\bar{\chi}(x)$$
$$+\sqrt{2}i\epsilon\sigma_{\mu\nu}\partial^\nu\xi(x) - \sqrt{2}i\bar{\epsilon}\bar{\sigma}_{\mu\nu}\partial^\nu\bar{\chi}(x)\Big\}$$

$$\Longrightarrow \delta(A_\mu(x)) = -\frac{i}{\sqrt{2}}\epsilon\partial_\mu\xi(x) + \frac{i}{\sqrt{2}}\partial_\mu\bar{\chi}(x)\bar{\epsilon} + \xi(x)\sigma_\mu\bar{\epsilon} + \epsilon\sigma_\mu\bar{\chi}(x)$$
$$+\sqrt{2}i\epsilon\sigma_{\mu\nu}\partial^\nu\xi(x) - \sqrt{2}i\bar{\epsilon}\bar{\sigma}_{\mu\nu}\partial^\nu\bar{\chi}(x)$$

$$\delta(\theta\theta\bar{\theta}\bar{\lambda}(x)) = -i\theta\sigma^\mu\bar{\theta}\,\partial_\nu A_\mu(x)\,\theta\sigma^\nu\bar{\epsilon} + i\theta\theta\,\partial_\mu M(x)\,\epsilon\sigma^\mu\bar{\theta}$$
$$+\frac{1}{2}\theta\theta\epsilon\bar{\theta}D(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\epsilon}D(x)$$

$$\left|\ \bar{\epsilon}\bar{\theta} = \bar{\theta}\bar{\epsilon} \ (I6), \quad \epsilon\sigma^\mu\bar{\theta} = -\bar{\theta}\bar{\sigma}^\mu\epsilon \ (I7a) \right.$$

$$= \theta\theta\bar{\theta}\bar{\epsilon}D(x) - i\theta\theta\bar{\theta}\bar{\sigma}^\mu\epsilon\,\partial_\mu M(x)$$
$$\underbrace{-i\theta\sigma^\mu\bar{\theta}(\theta\sigma^\nu)_{\dot{A}}}\bar{\epsilon}^{\dot{A}}\,\partial_\nu A_\mu(x)$$

$$\left|\ = +i(\theta\sigma^\nu)_{\dot{A}}\,\bar{\theta}\bar{\sigma}^\mu\theta\,\bar{\epsilon}^{\dot{A}} \quad \text{using} \quad \xi\sigma^\mu\bar{\chi} = -\bar{\chi}\bar{\sigma}^\mu\xi \ (I7a) \right.$$
$$\left|\ (\theta\sigma^\mu)_{\dot{A}}\,\bar{\theta}\bar{\sigma}^\nu\theta = -\theta\theta\left[\frac{1}{2}\bar{\theta}_{\dot{A}}g^{\mu\nu} + i(\bar{\theta}\bar{\sigma}^{\mu\nu})_{\dot{A}}\right] \ (I70) \right.$$

$(\theta\sigma^\nu)_{\dot{A}}\bar{\epsilon}^{\dot{A}}$ why index down and dotted on $\theta\sigma^\nu$? Not barr'd?

$$= \theta\theta\bar{\theta}\bar{\epsilon}D(x) - i\theta\theta\bar{\theta}\bar{\sigma}^\mu\epsilon\,\partial_\mu M(x)$$
$$+i\left(-\theta\theta\left[\frac{1}{2}\bar{\theta}_{\dot{A}}g^{\nu\mu} + i\underbrace{(\bar{\theta}\bar{\sigma}^{\nu\mu})_{\dot{A}}}_{\bar{\theta}_{\dot{B}}\bar{\sigma}^{\nu\mu\dot{B}}{}_{\dot{A}}}\right]\right)\bar{\epsilon}^{\dot{A}}\,\partial_\nu A_\mu(x)$$

rename $\mu\leftrightarrow\nu$

$$= \theta\theta\bar{\theta}_{\dot{A}}\Big\{\bar{\epsilon}^{\dot{A}}D(x) - i(\bar{\sigma}^\mu\epsilon)^{\dot{A}}\partial_\mu M(x) - \frac{i}{2}\bar{\epsilon}^{\dot{A}}\partial^\mu A_\mu(x)$$
$$+(\bar{\sigma}^{\mu\nu}\bar{\epsilon})^{\dot{A}}\partial_\mu A_\nu(x)\Big\}$$

$$\Longrightarrow \delta(\bar{\lambda}^{\dot{A}}(x)) = \bar{\epsilon}^{\dot{A}}D(x) - i(\sigma^\mu\epsilon)^{\dot{A}}\partial_\mu M(x) - \frac{i}{2}\bar{\epsilon}^{\dot{A}}\partial^\mu A_\mu(x)$$
$$+(\bar{\sigma}^{\mu\nu}\bar{\epsilon}^{\dot{A}})\partial_\mu A_\nu(x)$$

$$\delta(\bar\theta\bar\theta\theta\zeta(x)) = -i\,\bar\theta\bar\theta\,\partial_\mu N(x)\,\theta\sigma^\mu\bar\epsilon + i\,\theta\sigma^\mu\bar\theta\,\partial_\nu A_\mu(x)\,\epsilon\sigma^\nu\bar\theta + \frac{1}{2}\epsilon\theta\,\bar\theta\bar\theta D(x) + \frac{1}{2}\theta\epsilon\,\bar\theta\bar\theta D(x)$$

$$\epsilon\theta = \theta\epsilon \quad (\mathrm{I}6)$$

$$= -i\,\bar\theta\bar\theta\,\theta\sigma^\mu\bar\epsilon\,\partial_\mu N(x) + \bar\theta\bar\theta\,\theta\epsilon\, D(x)$$

$$\underbrace{+i\,\epsilon\sigma^\nu\bar\theta\;\theta\sigma^\mu\bar\theta}_{i\epsilon^A(\sigma^\nu\bar\theta)_A\,\theta\sigma^\mu\bar\theta}\,\partial_\nu A_\mu(x)$$

$$(\sigma^\mu\bar\theta)_A\,\theta\sigma^\nu\bar\theta = \bar\theta\bar\theta\left[\frac{1}{2}g^{\mu\nu}\theta_A - i(\sigma^{\mu\nu}\theta)_A\right] \quad (\mathrm{I}7a)$$

$$= \bar\theta\bar\theta\,\theta^A\left\{-i(\sigma^\mu\bar\epsilon)_A\,\partial_\mu N(x) + \epsilon_A D(x)\right\}$$
$$+ i\epsilon^A\left(\bar\theta\bar\theta\left[\frac{1}{2}g^{\nu\mu}\theta_A - i(\sigma^{\nu\mu}\theta)_A\right]\right)\partial_\nu A_\mu(x)$$

$$= \bar\theta\bar\theta\,\theta^A\Big\{-i(\sigma^\mu\bar\epsilon)_A\,\partial_\mu N(x) + \epsilon_A D(x)$$
$$+ \frac{i}{2}\epsilon_A\,\partial^\mu A_\mu(x) + \epsilon^A(\sigma^{\mu\nu}\theta)_A\,\partial_\mu A_\nu(x)\Big\}$$

where we used $\epsilon^A\theta_A = \epsilon\theta = \theta\epsilon = \theta^A\epsilon_A$ (I5),(I6)

$$\epsilon\sigma^{\mu\nu}\theta = -\theta\sigma^{\mu\nu}\epsilon = -\theta^A(\sigma^{\mu\nu}\epsilon)_A$$

$$= \bar\theta\bar\theta\,\theta^A\Big\{-i(\sigma^\mu\bar\epsilon)_A\,\partial_\mu N(x) + \epsilon_A D(x)$$
$$+ \frac{i}{2}\epsilon_A\,\partial^\mu A_\mu(x) - (\sigma^{\mu\nu}\epsilon)_A\,\partial_\mu A_\nu(x)\Big\}$$

$$\leadsto \delta(\zeta_A(x)) = -\frac{i}{2}(\sigma^\mu\bar\epsilon)_A\,\partial_\mu N(x) + \epsilon_A D(x) + \frac{i}{2}\epsilon_A\,\partial^\mu A_\mu(x)$$
$$- (\sigma^{\mu\nu}\epsilon)_A\,\partial_\mu A_\nu(x)$$

$$\delta\left(\frac{1}{2}\theta\theta\bar\theta\bar\theta D(x)\right) = i\,\theta\theta\bar\theta\,\partial_\mu\bar\zeta(x)\,\epsilon\sigma^\mu\bar\theta - i\,\bar\theta\bar\theta\theta\,\partial_\mu\zeta(x)\,\theta\sigma^\mu\bar\epsilon$$

$$\overset{(\mathrm{I}7a)}{=} -i\,\theta\theta\bar\theta\,\partial_\mu\bar\zeta(x)\,\bar\theta\bar\sigma^\mu\epsilon - i\,\bar\theta\bar\theta\theta\,\partial_\mu\zeta(x)\,\theta\sigma^\mu\bar\epsilon$$

$$\overset{(\mathrm{I}7c)}{=} -i\,\theta\theta\left\{-\frac{1}{2}\bar\theta\bar\theta\,\partial_\mu\bar\zeta(x)\,\bar\sigma^\mu\epsilon + \underbrace{\bar\theta\bar\sigma^{\mu\nu}\bar\theta}_{=0\,(\mathrm{I}7b)}\,\partial_\mu\bar\zeta(x)\,\bar\sigma_\nu\epsilon\right\}$$
$$-i\,\bar\theta\bar\theta\left\{-\frac{1}{2}\theta\theta\,\partial_\mu\zeta(x)\,\sigma^\mu\bar\epsilon + \underbrace{\theta\sigma^{\mu\nu}\theta}_{=0\,(\mathrm{I}7b)}\,\partial_\mu\zeta(x)\,\sigma_\nu\bar\epsilon\right\}$$

$$= \frac{i}{2}\theta\theta\bar\theta\bar\theta\left\{\partial_\mu\bar\zeta(x)\,\bar\sigma^\mu\epsilon + \partial_\mu\zeta(x)\,\sigma^\mu\bar\epsilon\right\}$$

$$\delta(D_\alpha) = i\partial_\mu \bar{\lambda}_\alpha \bar{\sigma}^\mu \epsilon + i\partial_\mu \zeta_\alpha \sigma^\mu \epsilon$$

$$= i\partial_\mu \left(\zeta_\alpha \sigma^\mu \epsilon + \bar{\lambda}_\alpha \bar{\sigma}^\mu \epsilon \right)$$