Disclaimer

The solution at hand was written in the course of the respective class at the University of Bonn. If not stated differently on top of the first page or the following website, the solution was prepared and handed in solely by me, Marvin Zanke. Anything in a different color than the ball pen blue is usually a correction that I or a tutor made. For more information and all my material, check:
https://www.physics-and-stuff.com/

I raise no claim to correctness and completeness of the given solutions! This equally applies to the corrections mentioned above.

This work by Marvin Zanke is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License.

Condensed Matter Exercise 5

28.06.16

1. Electrons in homogeneous Magnetic Fields

$\vec{E}=0$, hence the eq. of motion yields with $\vec{F}=\vec{F}_L$,

Why are those the trajectories in k-space!? lecture

$$m\ddot{\vec{r}} = -e(\vec{v}\times\vec{B}) = m\ddot{\vec{r}} = -e(\dot{\vec{r}}\times\vec{B})$$

Rotating our coordinates, so that $\vec{B}=B\vec{e_z}$ and

B const? yes

$$\vec{r}=\begin{pmatrix}x\\y\\z\end{pmatrix},\quad \dot{\vec{r}}=\begin{pmatrix}\dot{x}\\\dot{y}\\\dot{z}\end{pmatrix}$$

$$\Rightarrow \ddot{\vec{r}} = -\frac{e}{m}(\dot{\vec{r}}\times\vec{B}) = -\frac{eB}{m}(\dot{\vec{r}}\times\vec{e_z})$$

$$= -\frac{eB}{m}\begin{pmatrix}\dot{y}\\-\dot{x}\\0\end{pmatrix} = +\frac{eB}{m}(\dot{x}\vec{e_y}-\dot{y}\vec{e_x})$$

$\Rightarrow$ results in 3 coupled diff. eq.

CALCULATION LIKE THIS NOT VALID BECAUSE WE DO NOT CONSIDER BAND STRUCTURE

$$\ddot{x} = -\frac{e}{m}B\dot{y} \quad (1)$$

$$\ddot{y} = \frac{e}{m}B\dot{x} \quad (2)$$

$$\ddot{z} = 0 \quad (3)$$

(3) $\Rightarrow$ $z(t) = C_1t + C_2$, $C_i = \text{const}$, $i=1,2$

(1) $\Rightarrow$ $\dddot{x} = -\frac{e}{m}B\ddot{y} = -\frac{e^2}{m^2}B^2\dot{x}$

(2) $\Rightarrow$ $\dddot{y} = \frac{e}{m}B\ddot{x} = -\frac{e^2}{m^2}B^2\dot{y}$

$A\{\cos(ict)+i\sin(ict)\} + B\{\cos(ict)-i\sin(ict)\}$

$x(t)=e^{\lambda t} \Rightarrow \dot{x}(t)=\lambda e^{\lambda t}$ and $\dddot{x}(t)=\lambda^3 e^{\lambda t}$

$\overset{(1)}{\Rightarrow}$ $\lambda^3 e^{\lambda t} = -\frac{e^2}{m^2}B^2\lambda e^{\lambda t} \overset{\lambda\neq 0}{\Leftrightarrow} \lambda^2 = -\frac{e^2}{m^2}B^2$

Only real parts for exp.? (linear combination of sinus + cosinus, so that we get real part

$$\Rightarrow x(t) = Ae^{i\frac{eB}{m}t} + Be^{-i\frac{eB}{m}t}$$

Analogue $y(t) = Ae^{i\frac{eB}{m}t} + Be^{-i\frac{eB}{m}t}$

$z(t) = C_1t + C_2$

$$\Rightarrow r_\perp = \vec{r} - \hat{\vec{B}}(\hat{\vec{B}}\cdot\vec{r}) = \vec{r} - \vec{e_z}(\vec{e_z}\cdot\vec{r}) = \vec{r} - \vec{e_z}\,z(t)$$

$$= x(t)\vec{e_x} + y(t)\vec{e_y}$$

Board:

$$\frac{dk}{dt} = -\frac{e}{\hbar}\left(\frac{dr}{dt} \times \underline{B}\right)$$

$$\underline{k}(t) - k(0) = -\frac{e}{\hbar}\left(\underline{r}(t) - \underline{r}(0)\right) \times \underline{B}$$

$\underline{B} = B\vec{z}$

$$= -\frac{e}{\hbar}\left(\underline{r}(t) - r(0)\right)_{\perp} \times \underline{B}$$

$$x(t) - x(0) = \frac{\hbar}{eB}\left(k_y(t) - k_y(0)\right)$$

$$y(t) - y(0) = -\frac{\hbar}{eB}\left(k_x(t) - k_x(0)\right)$$

Without assuming $B = B\vec{z}$: $\underline{B}\times$ on both sides

$$\underline{B} \times \dot{k} = B \times \frac{-e}{\hbar}\left(\frac{dr}{dt} \times \underline{B}\right) = -eB\dot{r}_{\perp}$$

2. Classical theory of Cyclotron Resonance

isotropic effective mass?/ relaxation time?

$$m^* \dot{\vec{v}} + \frac{m^*}{\tau}\vec{v} = q(\vec{E} + \vec{v}\times\vec{B})$$

Ansatz: $\vec{E} = \vec{E}_0 e^{i\omega t}$, $\vec{E}_0 = \begin{pmatrix} E_x \\ 0 \\ 0 \end{pmatrix}$

Otherwise consider Bandstructure

$\vec{v}(t) = \vec{v}_0 e^{i\omega t} \Rightarrow \dot{\vec{v}}(t) = i\omega \vec{v}_0 e^{i\omega t}$, $\vec{v}_0 = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$

Ansatz?/ rel. time: time between scatters etc.

Driven by E-field with $e^{i\omega t}$

$\vec{B} = \begin{pmatrix} 0 \\ 0 \\ B_z \end{pmatrix}$

B-Field and e-m. wave (really a wave)! yes

yields: $i\omega \vec{v}_0 e^{i\omega t} + \frac{1}{\tau}\vec{v}_0 e^{i\omega t} = \frac{q}{m^*}\left(\vec{E}_0 e^{i\omega t} + (\vec{v}_0\times\vec{B})e^{i\omega t}\right)$

$$\Leftrightarrow \vec{v}_0\left(i\omega + \frac{1}{\tau}\right) = \frac{q}{m^*}\left(\vec{E}_0 + (\vec{v}_0\times\vec{B})\right)$$

$$\Leftrightarrow (1+i\omega\tau)\vec{v}_0 = \frac{q\tau}{m^*}\left(\vec{E}_0 + (\vec{v}_0 + \vec{B})\right)$$

Why neglect influence magn. field of e-m. wave

Super weak

$$\Leftrightarrow \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \frac{q\tau}{(1+i\omega\tau)m^*}\left[\begin{pmatrix} E_x \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} v_y B_z \\ -v_x B_z \\ 0 \end{pmatrix}\right]$$

$$v_x = \frac{q\tau'}{m^*}(E_x + v_y B_z)$$

$$v_y = -\frac{q\tau'}{m^*}v_x B_z \qquad \text{with } \tau' = \frac{\tau}{1+i\omega\tau}$$

$$\Rightarrow v_x = \frac{q\tau'}{m^*}\left(E_x - \frac{q\tau'}{m^*}v_x B_z^2\right)$$

$B \cong B_z$? in this case given ☐

$$= \frac{q\tau'}{m^*}E_x - \frac{q^2\tau'^2}{m^{*2}}v_x B_z^2$$

$$\Leftrightarrow v_x\left(1 + \frac{q^2\tau'^2}{m^{*2}}B_z^2\right) = \frac{q\tau'}{m^*}E_x$$

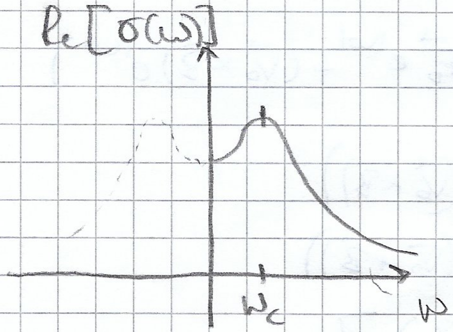
$$\Leftrightarrow v_x(1+\omega_c^2\tau'^2) = \frac{q\tau'}{m^*}E_x \qquad \text{with } \omega_c = \frac{qB}{m^*}$$

$$\sigma = \frac{j_x}{E_x} = \frac{Nqv_x}{E_x} = Nq\frac{q\tau'}{m^*}\left(\frac{1}{1+\omega_c^2\tau'^2}\right)$$

$$= \frac{Nq^2}{m^*}\frac{\tau}{1+i\omega\tau}\left(\frac{1}{1+\omega_c^2\frac{\tau^2}{(1+i\omega\tau)^2}}\right) = \frac{Nq^2\tau}{m^*}\left(\frac{1}{(1+i\omega\tau)+\omega_c^2\frac{\tau^2}{(1+i\omega\tau)}}\right)$$

$$= \frac{Nq^2\tau}{m^*}\frac{1+i\omega\tau}{(1+i\omega\tau)^2+\omega_c^2\tau^2} = \frac{Nq^2\tau}{m^*}\frac{1+i\omega\tau}{1+(\omega_c^2-\omega^2)\tau^2+2i\omega\tau}$$

$$\mathrm{Re}(\sigma) = \frac{1}{2}(\sigma + \sigma^*) = \frac{nq^2\tau}{2m^*} \frac{\left[2(\omega_c^2\tau^2 + \omega^2\tau^2 + 1)\right]}{\left[\omega_c^4\tau^4 - 2\omega_c^2\tau^4\omega^2 + 2\omega_c^2\tau^2 + \tau^4\omega^4 + 2\tau^2\omega^2 + 1\right]}$$

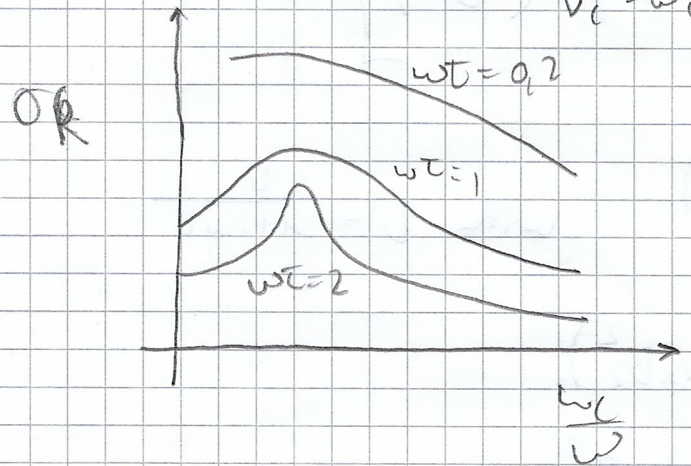


Nicer:

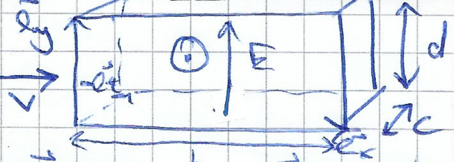
$$\frac{\sigma_R}{\sigma_0} = \frac{1 + \nu^2 + \nu_c^2}{(1 + \nu_c^2 - \nu^2)^2 + 4\nu^2}$$

$\nu = \omega\tau$ $\qquad \sigma_0 = \frac{Ne^2\tau}{m^*}$

$\nu_c = \omega_c\tau$



3. Hall effect



Instead $F_L = 0$? no force

Rotation so that fields in one direction always allowed?

a) $\vec{F_L} = q\vec{E} + q\vec{v} \times \vec{B} \overset{!}{=} 0$ with $\vec{B} = B\vec{e_z}$, $\vec{E} = E\vec{e_y}$, $\vec{v} = v\vec{e_x}$

$$\Rightarrow q E \vec{e_y} = -qvB \underbrace{(\vec{e_x} \times \vec{e_z})}_{-\vec{e_y}}$$

$$\Leftrightarrow E = vB \qquad \Big| E = \frac{U}{d}$$

$$\Leftrightarrow \frac{U}{d} = vB \qquad \Big| \cdot d, \cdot c$$

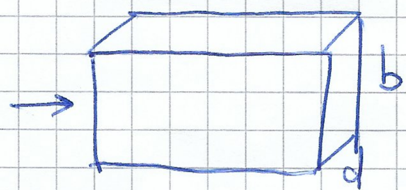
$$\Leftrightarrow U = c \cdot d \cdot vB \qquad \Big| A = cd, \quad I = neAv$$

Cross sectional area

$$\Leftrightarrow Uc = \frac{1}{ne} I B$$

$$\Leftrightarrow U_H = \frac{I}{nec} B$$

b)



$d = 0{,}2\,cm$, $b = 1\,cm$

$n = 10^{15} \frac{1}{cm^3}$

$B = 100\,mT$, $U_H = 10\,mV$

Volt meter sensitivity $\hat{=} U_H$? need at least this voltage

$$I = \frac{nec}{B} U_H = 32{,}04\ A$$

c) $R_H = \frac{1}{ne} \underbrace{\frac{\mu_h - \mu_e}{\mu_h + \mu_e}}_{\mu'}$, $U_H = R_H \frac{IB}{d}$

high temperatures: $n \approx n_i \gg N_D$ (intrinsic regime)

N_c, N_v: eff. density of e^-, holes

$$n_i = p_i = \sqrt{N_c N_v} \exp\left(-\frac{E_0}{2k_B T}\right)$$

intrinsic carrier concentration

$$R_H = \frac{1}{2e} \left(\frac{k_B T}{2\pi \hbar^2}\right)^{-3/2} (m_c m_v)^{-3/4} \exp\left(-\frac{E_0}{2k_B T}\right) \cdot \mu'$$

Plot

$(R_H) T^{3/2}$ vs $\frac{1}{T}$

$\log\left(|R_H| T^{3/2}\right) = \text{const} + \frac{E_g}{2k_B T}$

4. Properties of doped Silicon

$N_D = \frac{10^{16}}{cm^3}$, $n_i = 1{,}45 \cdot 10^{10} \frac{1}{cm^3}$, $N_C = 2{,}8 \cdot 10^{19} \frac{1}{cm^3}$

$T = 300K$

$$n \approx 2N_D \left(1 + \sqrt{1 + 4 \frac{N_D}{N_C} e^{\frac{E_d}{k_B T}}}\right)^{-1} \quad , \quad E_d = E_C - E_D$$

intermediate temperatures with $4 \frac{N_D}{N_C} e^{\frac{E_d}{k_B T}} \ll 1$: Saturation regime,

$n \approx N_D = \text{const} = 10^{16} \frac{1}{cm^3}$

$E_F(T) = E_C - k_B T \ln\left(\frac{N_C(T)}{N_D}\right)$, $E_C = 3{,}4eV$ (chapter 9, p. 2, fig. 9.1.5)

$\Rightarrow E_F = 3{,}40eV - 0{,}21eV = 3{,}19eV$