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https://www.physics-and-stuff.com/

I raise no claim to correctness and completeness of the given solutions! This equally applies to the corrections mentioned above.

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Glaneral Relativity 3. Exercise Homework Marin Earlie H6) a) would to prove a Rapper in + Rapper in + Rapper in =0 (5/5 30.04.2018 We recall that e.g. eille is wis Ruppir = 3x Paper - Repro Tax - Rueno Tex - Rapertax - Rappetra scally inextral Dord . Eystern 1 1 1/21 Shilon Where we lest out the explicit & objectiones of the Riemann Just in normal Evernamo Coord Jensor Rapper (X). like this? We now transform to a locally mentical coordisate sepseu for the fixed x, we want to prove the identity for. This will then be valid for all x, as x is arbitrary in this frame, the Christoffel-symbols voints at the origin, i.e. This (0) =0, Mat Is It While the dervative does not the centrally have to banish, Tps. 201 € here roll=0? possible to also It then suffices to prove Paper, 1 + Paper, v + Rapra, r = 0 Tare This =0 as all other terms vanish. We will give two proofs for this: First: 3xx Rappy + 3xx Rapxy + 3xx Rapxx Be cause you Canmake a = 3x (gae RBM) + 3x (gae RBXM) + 3x (gae RBVX) coordinate Inonedo in which Prv = Dxr TBv - Dxv Fpr + Tar FBv - Tav Fpr gand healty Der Tou Der Pr De vanishes Bat you cannot gaen Rom + gaen Pen + gaen Rom 5 nd one + gre / 3x (3x Len - 3x Len) + 2x (3x Le 3 Le) in gh which 9,29,229 M + 2x (3x 18y - 3x 18n) [voush, 1.2170. = gae, & Rpm + gae, v Rpip + gae, - Rpvi gapin = Dut grap - gep Kum - gae Pap = O coord. Dut gap = gapin

Publin = 2) 2xby gar + 2xxxx gp - 2xbx gar - 2xxx gpr + gro Jeb Lan - Lange = o a locally inertial coord. yeten WAS SXY BAPLA + SXX BABAY - 2 / 2x (2x 2x 2x 2x 2x 2pp - 3x 2x 2x 2pp)

- 2 / 2x (2x 2x 2x 2x 2x 2pp - 3x 2x 2x 2pp)

+ 2x (2x 2x 2x 2x 2x 2pp - 2x 2x 2x 2pp)

- 2 / 22 / 2x 2x 2x 2pp)

+ 2x (2x 2x 2x 2x 2x 2pp - 2x 2x 2x 2pp)

+ 2x (2x 2x 2x 2x 2x 2pp - 2x 2x 2x 2pp)

- 2 / 2x 2x 2x 2pp)

- 3 / 2x 2x 2x 2pp)

- 3x 2x 2x 2pp) = 5 Jorgangan dan + oxydagan dan - oxydagan dan - oxydagan dan Yes! But when you take derivative, you will have 20 Jems. But only with a P term. je 2(PP)=(2P)+ P(2P). thre for this term will not appear in set to zero! If the derivative.

C b) Equation (7) States Golin =0, where Godin = grad gris Rags - 2 grav R Sheeld: alle Rap = Raxp and R = grk Rpx of commute of Willing that for the covariant derivative (TW) 12-1/K p 1/2 - 1/2 / 1/2, soly iff vg = we have the product rule V(wow') = (vw) ou') + wo (vw') for a the metric? and thus will pull the metric inside covariant dervotives in this exercise, Violentes making use of 0= gdpin - (79) app. We will give two prob for Change back First : from a), we know the differential BIANCHA identity: to other coord. Reparix + Reparix + Reparin = 0 | x got (summerion included) dervaline now, Just a coordinade Rippinia - Rippinia + gar Rapunia = 0 Clasia = 0 DR(y) where we used Rpm x = g xt Rappur x = g xt Ra Duly for different andices?! (RBV; X - RBX; V + gat Rapul; r = 0 (x gpx) ("-----) Jas a doesn't same vda PARBUIL - RIV + gam R'alvip =0 at low. using R = gps Rps Bwt 8 B x = 0 = gpx PBVix - Riv + gBx RBVix =0 Yes. by renaving & -> B, p -> 1 in last term C=> gpl RBV; 1 = 2 Riv XgVk (=) gⁱβgkv Rβviλ = 2 gⁱR_iν | rename ν ↔ λ on r.h.s. (=) gⁱβgkv Rβviλ = 2 gⁱR_iλ (gⁱβgkv Rβviλ = 2 gⁱR)_{iλ} = 0 () GR = 0

Alternative (lainer?!) proof: 7 Golin = (grag of Rap - 2 gran R); r = grag of Rapin - 2 gran Rin = goodgip Rapir - 2 grugst RBAir - 2 grugpi Rpi, m = - 2 grugpi Reelin = - 2 grugpi ed Ropelin antisym 1 grugs Repris - 2 gruges Roxie

= -2 grugs Repris + georges Roxie

= -2 grugs Repris + georgia - 2 gruges Roxie

revone
= - grugs Repris + georgia - 2 gruges Roxie

revone
= - grugs Repris + georgia - 2 gruges Roxie

revone = gragin - gragin - grages Repris rename for chs.; fo > x, l >p, p > B = gragip Rapin - gragin =0