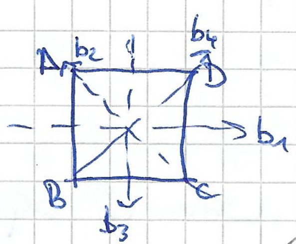
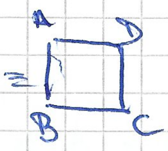
Disclaimer

The solution at hand was written in the course of the respective class at the University of Bonn. If not stated differently on top of the first page or the following website, the solution was prepared and handed in solely by me, Marvin Zanke. Anything in a different color than the ball pen blue is usually a correction that I or a tutor made. For more information and all my material, check:
https://www.physics-and-stuff.com/

I raise no claim to correctness and completeness of the given solutions! This equally applies to the corrections mentioned above.

21.10.2017 Group theory 2^{nd} Exercise — Marvin Zanke

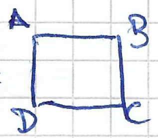
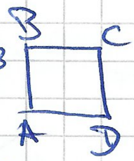
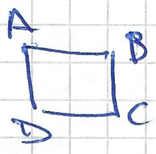
A2_1

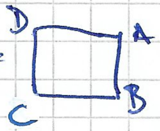
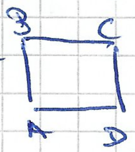
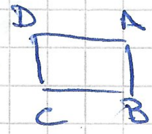
(1) Consider D_4,  , Let's denote $O =$ 

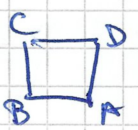
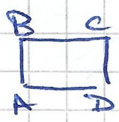
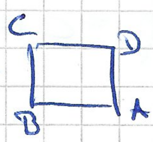
$c^{-1}bc = b_1 c^2$

We claim that: $b_2 = c^3 b_1$ $(= bc)$, $b_3 = c^2 b_1$, $b_4 = c b_1$ $(= bc^3)$ and show those relations on O.

Not really proven for D_n? thus can't claim D_n is generated by b and c?

$b_2 O =$  , while $c^3 b_1 O = c^3$  $=$ 

$b_3 O =$  , while $c^2 b_1 O = c^2$  $=$ 

$b_4 O =$  , while $c b_1 O = c$  $=$ 

(2)

$$D_n = \langle b, c \mid b^2 = c^N = (bc)^2 = e \rangle$$

Write as set $\{\dots\}$?

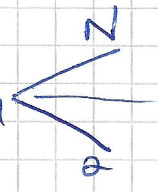
The order of D_n can easily be counted as:

$$\underbrace{b,\ b^2 = e,\ \underbrace{c, \dots, c^{N-1}}_{N-1}}_{N+1} + \underbrace{bc, \dots, bc^{N-1}}_{N-1} = 2N = |D_n|$$

As $(bc)^2 = e$, we find: $bc = (bc)^{-1} = c^{-1}b^{-1} = c^{n-1}b$

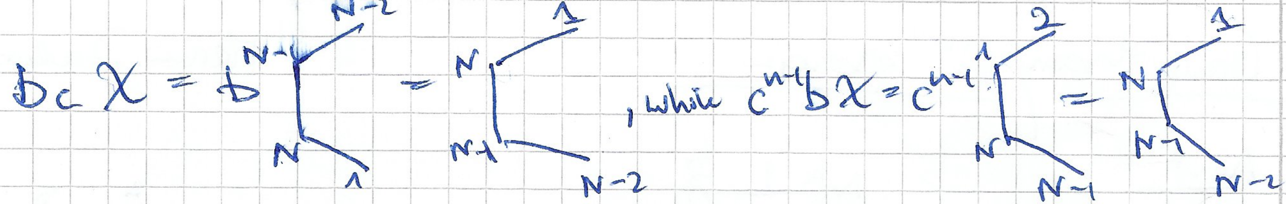
$$\Longleftrightarrow\ cbc = b \ \Leftrightarrow\ cb = bc^{n-1}$$

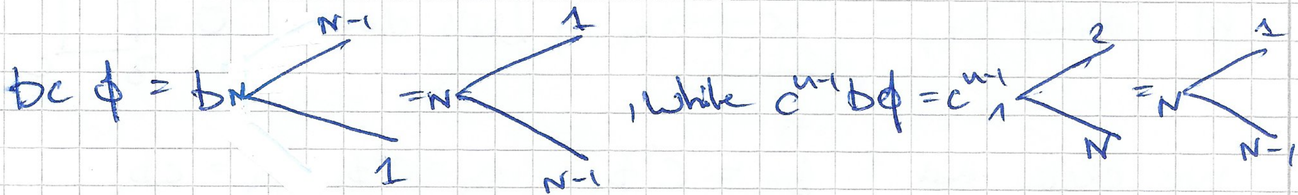
Why on polygons, not mathematically? Complex plane?

Let's denote $\chi \equiv$  , $\phi \equiv$  and distinguish

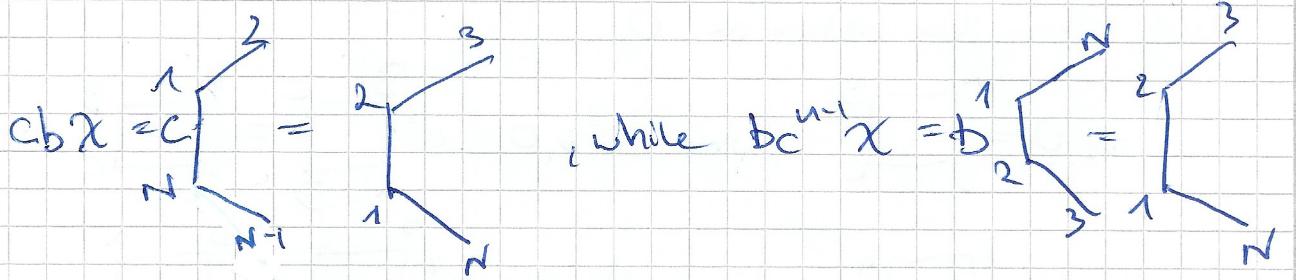
2 different cases?

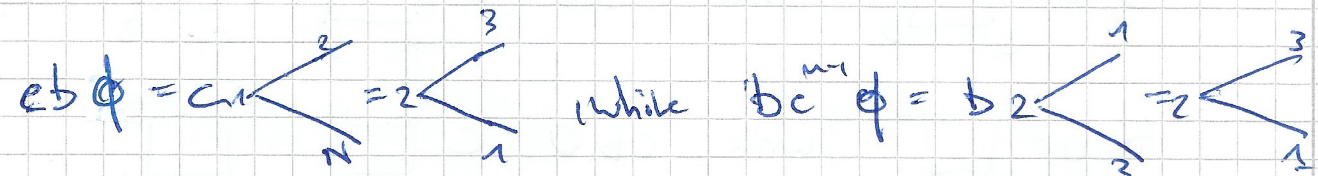
between the 2 cases, where the reflection axis cuts through a side and where the axis cuts between 2 sides (see χ and ϕ, respectively). We now show the reason to demand those conditions:





Enough to prove one of those identities, because the other is a consequence





(3)

With cycles? → also like this in test!

A2.2

(1)

	e	(12)	(13)	(23)	(123)	(132)
e	e	(12)	(13)	(23)	(123)	(132)
(12)	(12)	e	(132)	(123)	(23)	(13)
(13)	(13)	(123)	e	(132)	(12)	(23)
(23)	(23)	(132)	(123)	e	(13)	(12)
(123)	(123)	(13)	(23)	(12)	(132)	e
(132)	(132)	(23)	(12)	(13)	e	(123)

$(12)(23)(13)(32)$

$(123)(132) = (12)(23)(32)(21) = e$

$(132)(123) = (13)(32)(23)(31) = e$

order of $(j_1, \dots, j_k)$ is k

~~6 generaters~~ $S_3 = \langle (12)(23) \rangle$

2 generators

$(123) = (12)(23)$, $(132) = (123)(123) = (12)(23)(12)(23)$

$(13) = (123)(12) = (12)(23)(12)$

Not clear which order of S_N? → Yes, any group of order greater equal 6

(2) $$(1623)(3645)(1342) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 6 & 5 & 3 & 1 & 4 \end{pmatrix}$$
$$= (126435)$$

(*)

other way: start with (*) $(1) \to 3 \to 6 \to 2$, $2 \to 1 \to 6 \longrightarrow (126\ldots)$

Tride to calculate faster? → just calculate bravely here

$$(13)(413)(54321)(32)(135) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$
$$= (13)(13)(34)(1543)(32)(32)(135)$$
$$= (34)(4315)(135) = (34)(43)(315)(135)$$
$$= (315)(135) = (53)(31)(13)(35) = e$$

cycles commute, as no elements in common

(3) $$[(123456)(78910\,11)]^{30} = [(123456)]^{30}\,[(78910\,11)]^{30}$$
$$= e \cdot e = e, \text{ as } [(123456)]^6 = e$$
$$\text{and } [(78910\,11)]^5 = e$$

$$[\ldots]^{32} = \underbrace{(135)(246)}_{\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 1 & 2 \end{pmatrix}}(7\,9\,11\,8\,10)$$