Disclaimer

The solution at hand was written in the course of the respective class at the University of Bonn. If not stated differently on top of the first page or the following website, the solution was prepared and handed in solely by me, Marvin Zanke. Anything in a different color than the ball pen blue is usually a correction that I or a tutor made. For more information and all my material, check: 
https://www.physics-and-stuff.com/

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Group 6, \(16l \equiv N_a \), \(g \in G \), \(g^e \).

1. \(g^m = e\) \(\iff\) \(m = \min \{ k \mid g^k = e \}\).
   Construct a subgroup \(H = \langle g \rangle\), \(|i| = 1, ..., m\).
   \(\Rightarrow\) \(H = m\) and from Lagrange there:
   \(16l \mod 1H = 0\) and thus:
   \(N_a \mod m = 0 \Rightarrow \frac{N_a}{m} = c\).

2. Let's consider a group \(G\) with \(16l = p\) prime.
   We can choose \(e \in G\) and construct a subgroup
   \(H = \langle g \rangle, g^N = e\). We know \(|H| \geq 2\),
   but \(|H|\) needs to be a divisor of \(16l\) and thus \(|H| = p\).
   Thus \(G\) is generated by \(g\) and is cyclic.

   Example: \(Z_p\)

3. Groups with at most 4 elements are abelian.
   \(\Rightarrow\) \(Z_3\) as 3 prime has no proper subgroup.

4. \(H \subset G\), normal subgroup, \(gH = Hg\) \(\forall g \in G\).
   \(ghg^{-1} \in H\) for \(h \in H\).

   \(\Rightarrow\) \(ghg^{-1} = h\),
   \(g^{-1}h_1g = h_2 \Rightarrow g^{-1}h_2g = h_1 \Rightarrow g^{-1}h_1g = h_2 = g^{-1}g = g^{-1}h_2g = h_2 \Rightarrow\)
   \(gHg^{-1} = H\) as injectivity implies injectivity for equal cardinality.
\[(5) \quad (g_1 + H) (g_2 + H) = g_1 g_2 + H \quad \forall g_1, g_2 \in G \quad \text{will be G}
\]

Assume \( g' \) s.t. \( g' H + H g' \rightarrow (g'(1))(H + H(1))^{-1} \)
A.3.2
\[
\begin{align*}
    \mathbb{(H_n \times H_1)} / H_n & = \left\{ (x_i, y_i) + H_n \mid (x_i, y_i) \in (H_n \times H_2) \right\} \\
    \text{Map} \quad \beta \rightarrow \mathcal{A} \quad \sigma \rightarrow \begin{cases} 
        (x_i + b_i, a_i) \quad \rightarrow \quad (x_i + b_i, a_i) \\
        i = \ldots = m \quad \in H_n 
    \end{cases}
\end{align*}
\]

Isomorphism \( \sigma + H_2 \)

A.3.3

(1) \( Z(G) \) is abelian by construction and aribitrary associativity from \( G \).
- \( e \in Z(G) \) as \( eg = ge \) \( \forall g \in G \)
- \( a \in Z(G) \) \( \Rightarrow \) \( ag = ga \) \( \forall g \in G \) \( \Rightarrow g^{-1}ag = ag^{-1} \) \( \forall g \in G \)
- \( a, b \in Z(G) \) \( \Rightarrow \) \( ab = ba \) \( \forall g \in G \)

(2) \( g^{-2} \in Z(G) \) \( \forall g \in G \)
\[
(g \in G, h) = zh = hz = hg^{-1}h^{-1} \quad \text{is always trivial}
\]

(3) \( Z(G) \) proper. Assume \( G \) abelian \( \Rightarrow \) \( ab = ba \) \( \forall a, b \in G \)
\[
\Rightarrow Z(G) = G \quad \Rightarrow Z(G) \text{ not proper subgroup}
\]

(4)
\( H_1 = \frac{16 \sqrt{2}}{4} \Rightarrow \frac{16\sqrt{2}}{16} = 2 \)

\( \Rightarrow G = \underbrace{\sum_{G} g H}_{\text{geG}} = \underbrace{U}_{\text{geE}} H_g \)

\( = eH \underbrace{u}_{\text{geE}} H = \#eU H_g \text{ for } g \in H \)

\( \Rightarrow \text{weight } = H_g \text{ as } \text{lift} = \text{HL} \text{ for } \text{het minal} \)