Disclaimer

The solution at hand was written in the course of the respective class at the University of Bonn. If not stated differently on top of the first page or the following website, the solution was prepared and handed in solely by me, Marvin Zanke. Anything in a different color than the ball pen blue is usually a correction that I or a tutor made. For more information and all my material, check: https://www.physics-and-stuff.com/

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Group Theory Exercise 6

A6.1

Group G, \( D^6 \) a d-dimmedimensional representation and \( \{ V_i \}_1 \) a basis of the vector space. \( M \) an n-singular dxd matrix.

a) Subspace \( V'' \) spanned by \( \{ V_i \}_1 \), \( \{ V''_i \}_1 \), \( d_n < d \)

invariant under \( D^6_1 \), say

\[
D^6(g)(x, V_i + \ldots + k_n V_n) = \mu_n V_i + \ldots + \mu_n V_n
\]

Consider the representation \( D^6_1 = M^{-1}D^6_1 M \). Then

\( \{ V_i \}_1 \), \( \{ V''_i \}_1 \) is a basis of the vector space \( V'' \), which

is invariant under \( D^6_1 \), where \( M V_i = V'_i \Rightarrow V''_i = M^{-1}V_i \)

\[
D^6_1(g)(x, V_i + \ldots + k_n V_n)
\]

\[
= M^{-1}D(g)(x, V_i + \ldots + k_n V_n) M
\]

\[
= M^{-1}D(g)(x, V_i V_i + \ldots + k_n V_n)
\]

\[
= M^{-1}(\mu_n V_i + \ldots + \mu_n V_n)
\]

\[
= \mu_n V_i + \ldots + \mu_n V_n
\]

b) Let \( V = \sum_i a_i V_i \), and \( V'' = M(V_i) = \sum_{i=1}^{d} H_{ij} V_j \)

representation \( D^6_1(g) \) in basis \( \{ V_i \}_1 \) of \( V'' \):

\[
D^6_1(g)(V_i) = \sum_{k=1}^{d} (D^6(g))_{ki} V_k
\]

\[
\Rightarrow D^6_1(g)(V_i) = D^6(g) \left( \sum_{j=1}^{d} H_{ij} V_j \right) = \sum_{j=1}^{d} H_{ij} D^6(g)(V_j)
\]

\[
= \sum_{j=1}^{d} H_{ij} \sum_{k=1}^{d} (D^6(g))_{kj} V_k = \sum_{k=1}^{d} \sum_{j=1}^{d} H_{ij} (D^6(g))_{kj} V_k
\]

\[
= \sum_{k=1}^{d} \sum_{k=1}^{d} (D^6(g))_{ki} V_k
\]

\[
= \sum_{k=1}^{d} \sum_{k=1}^{d} (D^6(g))_{ki} V_k
\]

\[
= \sum_{k=1}^{d} \sum_{k=1}^{d} (D^6(g))_{ki} V_k
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\]

\[
= \sum_{k=1}^{d} \sum_{k=1}^{d} (D^6(g))_{ki} V_k
\]