

Disclaimer

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<https://www.physics-and-stuff.com/>

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Why no complex k. b. field?
↓

H7) $\mathcal{L} = \frac{1}{2} (\partial_\mu \phi(x) \partial^\mu \phi(x) - m^2 \phi^2(x)) + j(x) \phi(x)$
 $H_{\pm} = - \int d^3x j(x, t) \phi_{\pm}(x, t)$

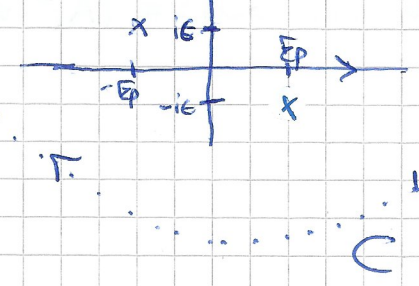
Hängt immer davon ab welche re. lds. du betrachten willst.
~~Effekt~~

First, we take a look at

Fourier transform of the current $j(x)$

$\lambda = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} |\tilde{j}(p)|^2$

$p^2 - m^2 + i\epsilon = (p^0)^2 - \vec{p}^2 - m^2 + i\epsilon = (p^0)^2 - E_p^2 + i\epsilon = (p^0)^2 - (E_p - i\epsilon)$
 $= (p^0 - (E_p - i\epsilon))(p^0 + (E_p - i\epsilon))$



Why close in lower arc? Not clear what $|\tilde{j}(p)|^2$ is and therefore whether it vanishes faster than p^2

Close integration contour in lower half, no contribution on arc for $|p^0| \rightarrow \infty$

$= \int \frac{d^3p}{(2\pi)^3} \int_{\mathcal{C}} \frac{dp^0}{2\pi} \frac{i}{(p^0 - (E_p - i\epsilon))(p^0 + (E_p - i\epsilon))} |\tilde{j}(p)|^2$

use residue theorem, negative winding number, pole of order 1 at $(E_p - i\epsilon)$

$= \int \frac{d^3p}{(2\pi)^3} i(-2\pi i) \left(\frac{1}{p^0 + (E_p - i\epsilon)} \right) \Big|_{p^0 = E_p - i\epsilon} |\tilde{j}(p)|^2 \Big|_{p^0 = E_p - i\epsilon}$

$\epsilon \rightarrow 0 \Rightarrow \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} |\tilde{j}(p)|^2$ with $p^0 = E_p$

Additionally

$\tilde{j}(p) = \int j(x) e^{-ipx} d^4x = \int d^4k j(k) e^{-ipk} d^4k = \int d^4k d^4l j(k) e^{-ipk} e^{ipl} = \int d^4k d^4l j(k) j(l) e^{i(p-l)x}$

$|\tilde{j}(p)|^2 = \tilde{j}(p) \cdot \tilde{j}^*(p) = \int d^4k d^4l j(k) j^*(l) e^{-ipk} e^{ipl} = \int d^4k d^4l j(k) j(l) e^{i(p-l)x}$

\uparrow j is real, well d.wg. Unitarität real sein muss

Now, finally:

derived in the lecture

Why the T m. A?

$K = \langle 0 | T \exp(-i \int dt H_{\pm}(t)) | 0 \rangle = \langle 0 | T \exp(+i \int d^4x j(x) \phi_{\pm}(x)) | 0 \rangle$
 $= \langle 0 | T \left\{ 1 + i \int d^4x j(x) \phi_{\pm}(x) - \frac{1}{2} \int d^4x j(x) \phi_{\pm}(x) \int d^4y j(y) \phi_{\pm}(y) + \mathcal{O}(j^3) \right\} | 0 \rangle$

$\approx \langle 0 | 0 \rangle + i \langle 0 | \int d^4x T j(x) \phi_{\pm}(x) | 0 \rangle - \frac{1}{2} \langle 0 | \int d^4x d^4y T j(x) \phi_{\pm}(x) j(y) \phi_{\pm}(y) | 0 \rangle$

$j(x)$ classical so T has no effect on it!

$= 1 + i \langle 0 | \int d^4x j(x) T \phi_{\pm}(x) | 0 \rangle - \frac{1}{2} \langle 0 | \int d^4x d^4y j(x) j(y) T \phi_{\pm}(x) \phi_{\pm}(y) | 0 \rangle$

$$= 1 + i \int d^4x j(x) \langle 0 | T \phi_{\pm}(x) | 0 \rangle - \frac{1}{2} \int d^4x d^4y j(x) j(y) \langle 0 | T \phi_{\pm}(x) \phi_{\pm}(y) | 0 \rangle$$

Wieder aus $\langle 0 | \dots | 0 \rangle$ ausrechnen?
 nur Zahlen, mit
 Klein OP \rightarrow Kommutator

$$= 1 + i \int d^4x j(x) \langle 0 | \phi_{\pm}(x) | 0 \rangle - \frac{1}{2} \int d^4x d^4y j(x) j(y) \langle 0 | \phi_{\pm}(x) \phi_{\pm}(y) + \phi_{\pm}(y) \phi_{\pm}(x) | 0 \rangle$$

as $\langle 0 | \dots | 0 \rangle$ always vanishes (normal ordered)

$$= 1 - \frac{1}{2} \int d^4x d^4y j(x) j(y) D_F(x-y) = 1 - \frac{1}{2} \int d^4x d^4y j(x) j(y) \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)}$$

$$= 1 - \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} \int d^4x d^4y j(x) j(y) e^{-ip(x-y)} = 1 - \frac{1}{2} \lambda$$

↑
see previous page

$$\rightarrow P(0 \rightarrow 0) = |A|^2 \approx \left| 1 - \frac{\lambda}{2} \right|^2 \approx 1 - \lambda$$



What for did we need $j(x) = 0$ for $t \rightarrow \pm \infty$?
 we need this for pert. theory

$$b) A = \langle 0 | T \sum_{n=0}^{\infty} \frac{(i \int d^4x j(x) \phi_{\pm}(x))^n}{n!} | 0 \rangle$$

$$= \sum_{n=0}^{\infty} \frac{i^n}{n!} \int d^4x_1 \dots d^4x_n \langle 0 | T j(x_1) \phi_{\pm}(x_1) \dots j(x_n) \phi_{\pm}(x_n) | 0 \rangle$$

$$= \sum_{n=0}^{\infty} \frac{i^n}{n!} \int d^4x_1 \dots d^4x_n j(x_1) \dots j(x_n) \langle 0 | T \phi_{\pm}(x_1) \dots \phi_{\pm}(x_n) | 0 \rangle$$

odd n
yield zero

$$\sum_{k=0}^{\infty} \frac{i^{2k}}{(2k)!} \int d^4x_1 \dots d^4x_{2k} j(x_1) \dots j(x_{2k}) \langle 0 | T \underbrace{\phi_{\pm}(x_1) \dots \phi_{\pm}(x_{2k})}_{\text{Pairs of propagators}} | 0 \rangle$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \int d^4x_1 \dots d^4x_{2k} j(x_1) \dots j(x_{2k}) \sum_{\text{pairings of fields } i} D_F(x_{i_1} - x_{i_2}) \dots D_F(x_{i_{2k-1}} - x_{i_{2k}})$$

Why do all
summands yield
the same
and therefore
 $\sum \rightarrow k!$?

Summing over this sum of products has

$$\frac{(2k)!}{(2k-1)!2!} \frac{(2k-2)!}{(2k-4)!2!} \dots \frac{2!}{2!0!} = \frac{(2k)!}{(2!)^k}$$

Summands, which all yield the

same, as it is symmetric in i . As the order we choose those
pairs out of $\phi_{\pm}(x_1) \dots \phi_{\pm}(x_{2k})$ is not important, we need to
factor out $\frac{1}{k!} \rightarrow \frac{(2k)!}{(2!)^k k!}$ ✓

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \int d^4x_1 \dots d^4x_{2k} j(x_1) \dots j(x_{2k}) \frac{(2k)!}{(2!)^k k!} D_F(x_1 - x_2) \dots D_F(x_{2k-1} - x_{2k})$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2!)^k k!} \underbrace{\int d^4x_1 j(x_1) j(x_2) D_F(x_1 - x_2)}_{\lambda} \dots \underbrace{\int d^4x_{2k} j(x_{2k-1}) j(x_{2k}) D_F(x_{2k-1} - x_{2k})}_{\lambda}$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{1}{2^k} \lambda^k = \sum_{k=0}^{\infty} \left(\frac{-\lambda}{2} \right)^k \frac{1}{k!} = e^{-\lambda/2}$$

$$\hookrightarrow P(0 \rightarrow 0) = |A|^2 = \left(e^{-\lambda/2} \right)^2 = e^{-\lambda}$$

