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String Theory Exercise 5 Homework

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$5+5+5+10 = 25$

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1) Mass shell condition for a state $|\psi\rangle$: $(\underset{\text{N.O. op.}}{L_0} - \underset{\text{const.}}{a})|\psi\rangle = 0$

Physical states: Mass shell condition + $L_m|\phi\rangle = 0 \ \forall m > 0$

Spurious state: Orthogonal to all phys. states $\langle\Psi|\phi_i\rangle = 0 \ \forall i$

Note that if a state is spurious and physical, it has zero norm $\langle\Psi|\Psi\rangle = 0$, because per definition it is orthogonal to all physical states, while being a physical state itself. ($\rightsquigarrow$ null state)

First excited state in open string theory $|\zeta, k\rangle = \underset{\text{pol. vector}}{\zeta} \cdot \alpha_{-1}|0,k\rangle$

vacuum, i.e. unexcited string w/ momentum k^μ

$\rightsquigarrow$ Mass $M^2 = \frac{1}{l_s^2}(N-a)$

$N = \sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n$ number operator

1. We calculate

$$[N, \alpha^\mu_{-n_i}] = \sum_{n=1}^{\infty}[\alpha_{-n}\alpha_n, \alpha^\mu_{-n_i}] = \sum_{n=1}^{\infty}\Big\{\eta_{\nu k}\alpha^\nu_{-n}[\alpha^k_n, \alpha^\mu_{-n_i}] + \eta_{\nu k}\underbrace{[\alpha^k_{-n}, \alpha^\mu_{-n_i}]}_{=0 \text{ for } n>0,\, n_i>0}\alpha^\nu_n\Big\}$$

$n_i > 0$

$$= \sum_{n=1}^{\infty}\alpha^\mu_{-n}\,\delta_{n-n_i,0}\, n = n_i\,\alpha_{-n_i}$$

$$\rightsquigarrow N\alpha^\mu_{-n_i} = \alpha^\mu_{-n_i}N + \alpha^\mu_{-n_i}n_i = \alpha^\mu_{-n_i}(N+n_i)$$

Then, it's easy to calculate

Momentum k and no. of α's also k? $\rightsquigarrow$ yes, but nothing to do w/ each other ✓

$$N\alpha_{-n_1}\cdots\alpha_{-n_k}|0,k\rangle = \alpha_{-n_1}(N+n_1)\alpha_{-n_2}\cdots\alpha_{-n_k}|0,k\rangle$$

$$= \ldots = \alpha_{-n_1}\cdots\alpha_{-n_k}\Big(N+\sum_{i=1}^{k}n_i\Big)|0,k\rangle$$

No contractions on α's? $\rightsquigarrow$ No, for fixed indices

✓ $N|0,k\rangle = 0$

$$\Big(\sum_{i=1}^{k}n_i\Big)\alpha_{-n_1}\cdots\alpha_{-n_k}|0,k\rangle$$

2. Consider $|\psi\rangle = L_{-1}|0,k\rangle$. We want to show that this state is spurious, i.e. $\langle\phi_i|\psi_i\rangle = 0 \;\forall\; |\phi_i\rangle$ physical. (and $(L_0 - a)|\psi\rangle = 0$).

$$\langle\phi_i|\psi\rangle = \langle\phi_i|L_{-1}|0,k\rangle = \underbrace{\langle\phi_i|L_1^\dagger}_{=(L_1|\phi_i\rangle)^* = 0}|0,k\rangle = 0$$

✓ Not supposed to show $(L_0 - a)|\psi\rangle = 0$ for spurious state $\rightsquigarrow$ depends on def. w/o cond.: D-branes by Clifford Johnson, Polchinski, and w/ cond.: Green Schwartz-Witten

$$L_0|\psi\rangle = L_0 L_{-1}|0,k\rangle = (L_{-1}L_0 + L_{-1})|0,k\rangle$$
$$= L_{-1}(L_0 + 1)|0,k\rangle = L_{-1}(a+1)|0,k\rangle$$

Doesn't satisfy mass-shell condition $\rightsquigarrow$ a "spurious state" w/ respect to different convention

3. $$M^2|\zeta,k\rangle = \frac{1}{\ell_s^2}(N-a)\zeta\cdot\alpha_{-1}|0,k\rangle = \frac{1}{\ell_s^2}(N-a)\zeta^\mu\alpha_{-1}^\nu\eta_{\mu\nu}|0,k\rangle$$

$\Big|\; [N,\alpha_{-1}^\nu] = \alpha_{-1}^\nu$ from 1.1)

$$= \frac{1}{\ell_s^2}\alpha_{-1}^\nu(N+1-a)\zeta^\mu\eta_{\mu\nu}|0,k\rangle$$

$N|0,k\rangle = 0$

$$= \frac{1}{\ell_s^2}\alpha_{-1}^\nu(1-a)\zeta^\mu\eta_{\mu\nu}|0,k\rangle$$
$$= \frac{1}{\ell_s^2}(1-a)\zeta\cdot\alpha_{-1}|0,k\rangle$$

$\rightsquigarrow$ ~~$a<1$~~ : $a>1$: negative mass state, tachyon; $a=1$: zero mass mode; ~~$a>1$~~ : $a<1$: positive mass

Consider for $m>0$: $\quad (L_0 - a)L_{-1}|0,k\rangle = (-k^2 + 1 - a)L_{-1}|0,k\rangle$

$$L_m|\psi\rangle = L_m L_{-1}|0,k\rangle = \Big(L_{-1}L_m + (m+1)L_{m-1} + \underbrace{\frac{D}{12}(m^3-m)\delta_{m-1,0}}_{=0}\Big)|0,k\rangle$$

$L_m|0,k\rangle = 0$

$$= (m+1)L_{m-1}|0,k\rangle = \begin{cases} 0 & \text{for } m\neq 1 \\ 2L_0|0,k\rangle = 2a|0,k\rangle & \text{for } m=1. \end{cases}$$

For a massless vector in the spectrum, we need $a=1$. Then $|\psi\rangle$ not physical.

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But the state is not physical then and thus not in the spectrum?

4. $L_n |\xi, k\rangle \overset{!}{=} 0$ for $n>0$

We first calculate (with $L_m = \frac{1}{2}\sum_{n\neq 0} :\alpha_{m-n}\alpha_n:$):

Decouple from physical processes – why (because orth.?) ⇝ they do not appear in the spectrum because we identify them. ✓

$$[L_m, \alpha_{-1}^\nu] = \frac{1}{2}\Big\{\sum_{n=-\infty}^{0}[\alpha_n\alpha_{m-n}, \alpha_{-1}^\nu] + \sum_{n=1}^{\infty}[\alpha_{m-n}\alpha_n, \alpha_{-1}^\nu]\Big\}$$

$$= \frac{1}{2}\Big\{\sum_{n=-\infty}^{0}\alpha_n^\lambda \eta_{\lambda k}[\alpha_{m-n}^k, \alpha_{-1}^\nu] + \eta_{\lambda k}\underbrace{[\alpha_n^\lambda, \alpha_{-1}^\nu]}_{=0 \text{ for } n<0}\alpha_{m-n}^k$$

$$+ \sum_{n=1}^{\infty}\alpha_{m-n}^\lambda[\alpha_n^k, \alpha_{-1}^\nu]\eta_{\lambda k} + [\alpha_{m-n}^\lambda, \alpha_{-1}^\nu]\alpha_n^k\eta_{\lambda k}\Big\}$$

$$= \frac{1}{2}\Big\{\sum_{n=-\infty}^{0}\alpha_n^\lambda \eta_{\lambda k}\eta^{k\nu}\delta_{m-n-1,0}(m-n)$$

$$+ \sum_{n=1}^{\infty}\alpha_{m-n}^\lambda \eta_{\lambda k}\eta^{k\nu}\delta_{n-1,0}\, n + \eta_{\lambda k}\eta^{\lambda\nu}\delta_{m-n-1,0}(m-n)\alpha_n^k\Big\}$$

$$= \frac{1}{2}\Big\{\alpha_{m-1}^\nu(m-1) + \alpha_{m-1}^\nu + \alpha_{m-1}^\nu(m-1)\Big\}$$

$$= \alpha_{m-1}^\nu \cdot m - \frac{1}{2}\alpha_{m-1}^\nu = \alpha_{m-1}^\nu\left(m - \frac{1}{2}\right)$$

Then $L_n|\xi, k\rangle = L_n \xi^\mu \alpha_{-1}^\nu \eta_{\mu\nu}|0,k\rangle$

$$= \xi^\mu\left(\alpha_{-1}^\nu L_n + \alpha_{n-1}^\nu\left(n-\tfrac{1}{2}\right)\right)\eta_{\mu\nu}|0,k\rangle$$

$$= \xi\cdot\alpha_{-1}\underbrace{L_n|0,k\rangle}_{=0 \text{ for } n\geq 1} + \xi\cdot\underbrace{\alpha_{n-1}\left(n-\tfrac{1}{2}\right)|0,k\rangle}_{=0 \text{ for } n>1}$$

$$= \begin{cases} 0 & \text{for } n>1 \\ \frac{1}{2}\xi\cdot\alpha_0|0,k\rangle \sim \xi_\mu \cancel{k^\mu}|0,k\rangle & \text{for } n=1 \end{cases}$$

$\hat{\alpha}_0 \sim \hat{k}$

5. Having $0 = k^\mu\xi_\mu|0,k\rangle = \int d^dx\, k^\mu\xi_\mu|0,x\rangle\langle 0,x|0,k\rangle$

Is it $|0,x\rangle$ or just $|x\rangle$?

$$= \int d^dx\, k^\mu\xi_\mu e^{-ikx}|0,x\rangle = \int d^dx\, i\xi_\mu(\partial^\mu e^{-ikx})|0,x\rangle$$

±i in exp.?

$$\overset{\text{int. by parts}}{=} -i\int d^dx\, \xi_\mu(\partial^\mu|0,x\rangle)e^{-ikx}$$

✗

⇝ $\xi_\mu\partial^\mu|0,x\rangle = 0$ or $\xi_\mu\partial^\mu \hat{=} 0$

$\partial_\mu\xi^\mu(x)$

6. $L_{-1}|0,k\rangle = \frac{1}{2}\sum_{n\neq 0} :\alpha_{-1-n}\alpha_n: |0,k\rangle = \frac{1}{2}\left\{\sum_{n=-\infty}^{0}\alpha_n\alpha_{-1-n} + \sum_{n=1}^{\infty}\alpha_{-1-n}\underbrace{\alpha_n\}|0,k\rangle}_{=0}$

$\left|\begin{matrix} -1-n\leq 0 \Leftrightarrow -1\leq n \\ \wedge\ n\leq 0 \end{matrix}\right\}\ n=-1 \wedge n=0$ possible

$= \frac{1}{2}\{\alpha_0\alpha_{-1} + \alpha_{-1}\alpha_0\}|0,k\rangle = \alpha_0\alpha_{-1}|0,k\rangle$

$[\alpha_0,\alpha_{-1}]=0$

$\Big|\ \alpha_{0\mu} = \sqrt{2\alpha'}k_\mu$

$= \sqrt{2\alpha'}\cancel{k_\mu\alpha^\mu_{-1}}|0,k\rangle$

Before: $k_\mu = {}^2/_{l_s}a_{0\mu}$ and $l_s = 2\pi\sqrt{\alpha'}$ Where is π?

7. Having $|\phi\rangle = \int d^dx\,\phi(x)|0,x\rangle$.

$\sim |\phi\rangle + |\eta\rangle = \int d^dx\,(\phi(x)+\eta(x))|0,x\rangle$ For $|\eta\rangle$ null state

$\rightsquigarrow \int d^dx\,\eta(x)|0,x\rangle = 0$ X

Insert (11)?
$\rightsquigarrow$ See tutorial

$|\phi\rangle \sim |\phi\rangle + |\eta\rangle$

$\xi^\mu \sim \xi^\mu + \lambda k^\mu$

$\xi^\mu(x) \sim \xi^\mu(x) + \partial^\mu\lambda$.

2) Consider $|2_f\rangle = L_{-1}|\chi_1\rangle$ where $(L_0 - a + 1)|\chi_1\rangle = 0$

$L_m|\chi_1\rangle = 0,\ m>0$

1. Physical state condition $L_m|2_f\rangle = 0$ for $m>0$

why $(L_0-a+1)|\chi_1\rangle = 0$? In particular for $m=1$:

$$L_1|2_f\rangle = L_1 L_{-1}|\chi_1\rangle = \Big(L_{-1}L_1 + 2L_0 + \underbrace{\tfrac{D}{12}(m^3-m)\delta_{m,1}}_{=0}\Big)$$

Why demand for especially these constructed state(s) that they are physical?

$$\overset{L_1|\chi_1\rangle=0}{=} \cancel{2(a-1)}|\chi_1\rangle \overset{!}{=} 0$$

$\rightsquigarrow a = 1$

2. Now consider $|2_f\rangle = (L_{-2} + \gamma L_{-1}^2)|\bar{\chi}\rangle$

where $\underbrace{(L_0+1)}_{(L_0-a+n)}|\bar{\chi}\rangle = 0;\ L_m|\bar{\chi}\rangle = 0,\ m>0$

only use this when fulfilled

Then with $X_m := \frac{D}{12}(m^3-m)\delta_{m+m_0}$

$$0 \overset{!}{=} L_1|2_f\rangle = L_1(L_{-2}+\gamma L_{-1}^2)|\bar{\chi}\rangle$$

$$= (L_{-2}L_1 + 3L_{-1})|\bar{\chi}\rangle + \gamma(L_{-1}L_1 + 2L_0 + X_1)L_{-1}|\bar{\chi}\rangle$$

$$\overset{L_1|\bar{\chi}\rangle=0,\ X_1=0}{=} 3L_{-1}|\bar{\chi}\rangle + \gamma(L_{-1}^2L_1 + L_{-1}(2L_0+X_1) + 2L_{-1}L_0 + 2L_{-1})|\bar{\chi}\rangle$$

$$\overset{X=0,\ L_1|\bar{\chi}\rangle=0,\ L_0|\bar{\chi}\rangle=-|\bar{\chi}\rangle}{=} 3L_{-1}|\bar{\chi}\rangle + \gamma(-2L_{-1} + 2L_{-1} - 2L_{-1})|\bar{\chi}\rangle$$

$$= (3-2\gamma)L_{-1}|\bar{\chi}\rangle \rightsquigarrow \gamma = 3/2$$

$$L_2|2_f\rangle = L_2(L_{-2}+\gamma L_{-1}^2)|\bar{\chi}\rangle$$

$$= (L_{-2}L_2 + 4L_0 + X_2)|\bar{\chi}\rangle + \gamma(L_{-1}L_2 + 3L_1)L_{-1}|\bar{\chi}\rangle$$

$$\overset{L_2|\bar{\chi}\rangle=0}{=} (4L_0+X_2)|\bar{\chi}\rangle + \gamma(L_{-1}^2L_2 + L_{-1}\cdot(3L_1) + 3L_{-1}L_1 + 3(2L_0+X_1))$$

$$\overset{X_1=0,\ L_1|\bar{\chi}\rangle=0,\ L_2|\bar{\chi}\rangle=0,\ L_0|\bar{\chi}\rangle=-|\bar{\chi}\rangle}{=} (4L_0+X_2)|\bar{\chi}\rangle + \cancel{6\gamma L_0|\bar{\chi}\rangle} = -13|\bar{\chi}\rangle + \tfrac{D}{12}(6)$$

$$= \left(\tfrac{D}{2} - 13\right)|\bar{\chi}\rangle$$

In order for $L_2|2_f\rangle \overset{!}{=} 0 \rightsquigarrow D = 26$

Meromorphic?

3) $\zeta(s) = \sum_{m\geq 1}^{\infty} \frac{1}{m^s}$, $\mathrm{Re}\, s > 1$ Can be analytically continued to a meromorphic function on the whole complex plane with $\zeta(-1) = -\frac{1}{12}$

a) Looking at the not-N.O. L_0, we find:

In L_0 summed over i? Metric for $i \in 1, \dots, D-2$ only positive $\Rightarrow$?

$$L_0 = \frac{1}{2} \sum_{n\neq 0} \alpha_{-n} \cdot \alpha_n \overset{\substack{\text{Sum only over } i = 1,\dots,D-2 \\ \text{where metric} = +1}}{=} \frac{1}{2} \left\{ \sum_{n=-\infty}^{0} \alpha_{-n}^i \alpha_n^i + \sum_{n\geq 1}^{\infty} \alpha_{-n}^i \alpha_n^i \right\}$$

$$= \frac{1}{2} \left\{ \sum_{n=-\infty}^{0} \left(\alpha_n^i \alpha_{-n}^i + [\alpha_{-n}^i, \alpha_n^j] \delta_{ij} \right) + \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i \right\}$$

Term to (−) term because of the n in term from comm.?

$$= \frac{1}{2} \sum_{n\neq 0} :\alpha_{-n} \cdot \alpha_n: + \frac{1}{2} \sum_{n=-\infty}^{0} [\alpha_{-n}^i, \alpha_n^j] \delta_{ij}$$

$$\overset{n \to -n}{=} L_0^{N.O.} + \frac{1}{2} \sum_{n=0}^{\infty} [\alpha_n^i, \alpha_{-n}^j] \delta_{ij}$$

$$\overset{\alpha_0 \sim p,\ [p^i, p^j] = 0}{=} L_0^{N.O.} + \frac{1}{2} \sum_{n\geq 1}^{\infty} [\alpha_n^i, \alpha_{-n}^j] \delta_{ij}$$

$$\overset{!}{=} L_0^{N.O.} - a \quad \cancel{\Rightarrow} \quad a = -\frac{1}{2} \sum_{n\geq 1}^{\infty} [\alpha_n^i, \alpha_{-n}^j] \delta_{ij}$$

✓ Why is this now $-a$? Starting from $L_0^{N.O.}$ we find the opposite? Or could repeat the step? $\Rightarrow$ yes; want to express L_0 in terms of $L_0^{N.O.}$, so start from L_0.

b) Calculating:

$$a = -\frac{1}{2} \sum_{n\geq 1}^{\infty} [\alpha_n^i, \alpha_{-n}^j] \delta_{ij} = -\frac{1}{2} \sum_{n\geq 1}^{\infty} n\, \delta^{ij} \delta_{ij}$$

$$= -\frac{1}{2} (d-2) \sum_{n\geq 1}^{\infty} n \quad \text{as only } d-2 \text{ oscillation modes in light cone gauge quantization}$$

$\zeta(-1) = -\frac{1}{12}$

But α_- too?

$$= \frac{1}{24} (d-2).$$

We already found $a = 1$ $\cancel{\Rightarrow}$ $d = 26$!

4) Open bosonic string theory: $(L_0 - a)|\phi\rangle = 0$ (NN B.C.)

$$\alpha' M^2 = 2p^+p^- - \sum_{i=1}^{24} p_i^2 = N - 1$$

Closed bosonic string theory: $(L_0 - a)|\phi\rangle = 0$

$$(\bar{L}_0 - a)|\phi\rangle = 0$$

$$\alpha' M^2 = 4(N-1) = 4(\bar{N}-1)$$

where $N = \sum_{i=1}^{24}\sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i$, $\bar{N} = \sum_{i=1}^{24}\sum_{n=1}^{\infty} \bar{\alpha}_{-n}^i \bar{\alpha}_n^i$

a) For the open bosonic string spectrum, the first three levels are:

$\underline{N=0}$: $|0,p\rangle$, the ground state. It's a ~~1-dimensional~~ scalar (1)

"corresponding to their SO(24) repr." ?

$\underline{N=1}$: $\alpha_{-1}^i|0,p\rangle$, first excited state, 24-dim. as $i = 1,\dots,d-2$ with $d=26$, i.e. 24 states. $\square_{24}$ $\rightsquigarrow$ $\underline{\underline{24}}$

vector

$\underline{N=2}$: $\alpha_{-2}^i|0,p\rangle \rightsquigarrow \square_{24}$

$\alpha_{-1}^i\alpha_{-1}^j|0,p\rangle$ is a symmetric tensor of rank 2. splits in $\underline{\text{sym. traceless tensor + trace}}$ (1)

Why split in $i \neq j$ and $i = j$ in lecture?

$\frac{24 \cdot 25}{2}$ $\xrightarrow{\text{these}}$ $\boxed{24}\boxed{25}$ $\hat{=}$ $300 = \underline{\underline{299 + 1}}$ $\rightsquigarrow$ $\underline{\underline{324}}$

$\underline{N=3}$: $\alpha_{-3}^i|0,p\rangle \rightsquigarrow \underline{\square_{24}} \rightsquigarrow \underline{\underline{24}}$

$\alpha_{-2}^i\alpha_{-1}^j|0,p\rangle$ is $\underline{\text{mixed symmetric}}$ tensor of rank 2. this splits into $\underline{\text{sym.}}$ and $\underline{\text{antisym. part}}$.

What about $\alpha_{-1}^j\alpha_{-2}^i$ - not the same state?

$\begin{smallmatrix}\boxed{24}\\\boxed{23}\end{smallmatrix}$ $\hat{=}$ $\underline{276}$ + $\boxed{24}\boxed{25}$ $\hat{=}$ $\underline{300 = 299 + 1}$

So e.g. 276 is the dim. of the rep. for the antisym. tensor in SO(24)?

?

$\alpha_{-1}^i\alpha_{-1}^j\alpha_{-1}^k|0,p\rangle$ symmetric tensor of rank 3

$\boxed{24}\boxed{25}\boxed{26}$ $\hat{=}$ $\underline{2600}$ $\rightsquigarrow$ $\underline{3200}$ $\underline{\text{total}}$

b) For the masses, we use $\alpha' M^2 = N - 1$ and that massless states fall into $SO(24)$, while massive into $SO(25)$.

$\underline{N=0}$: $\alpha' M^2 = -1 \leadsto M^2 < 0 \leadsto$ LG $SO(25)$

$\underline{N=1}$: $\alpha' M^2 = 0 \leadsto M^2 = 0 \leadsto$ LG $SO(24)$

$\underline{N=2}$: $\alpha' M^2 = 1 \leadsto M^2 > 0 \leadsto$ LG $SO(25)$

$\underline{N=3}$: $\alpha' M^2 = 2 \leadsto M^2 > 0 \leadsto$ LG $SO(25)$

What LG for neg. M^2?

For the number of states corresponding to their LG rep., we find

$\underline{N=0}$: trivial rep., 1-dim. $\leadsto \underline{1}$ in $\underline{SO(25)}$

$\underline{N=1}$: 24-dim. rep. as it's in $SO(24)$, equal to before $\leadsto \underline{24}$ in $\underline{SO(24)}$

$\underline{N=2}$: in $\underline{SO(25)}$: $\boxed{25}\boxed{26} \mathrel{\hat{=}} 325 = \underline{324} + 1 \leadsto \underline{\underline{324}}$

symmetric and traceless

No antisym. possible for N=2?

$\underline{N=3}$: in $\underline{SO(25)}$: $\boxed{25}\boxed{26}\boxed{27} + \begin{array}{|c|}\hline 25 \\ \hline 26 \\ \hline\end{array} \mathrel{\hat{=}} 2925 + 300$

$= (\underline{2900} + 25) + \underline{300}$

symmetric, traceless and antisymmetric

rank 3 $\leadsto$ trace × dim

N=2 in SO(25) a 324 dim rep. now?

$\leadsto \underline{\underline{3200 \text{ total}}}$?

Note: The question only asks the first 3 levels. Beyond that, I haven't checked. Sorry

Including $|0,k\rangle$.

3) $(L_0 - a)|\phi\rangle = (\bar{L}_0 - a)|\phi\rangle = 0$ implies that $N = \bar{N}$ for the closed string.

But only in total, not for each state

d) For the closed string, the first two levels are:

$N = \tilde{N} = 0$: $|0,p\rangle$, the ground state $\quad (1) \rightsquigarrow \underline{1}$

Also w/ $\alpha_{-1}^i \tilde{\alpha}_{-1}^j$ exchanged?

$N = \tilde{N} = 1$: $\alpha_{-1}^i \tilde{\alpha}_{-1}^j |0,p\rangle$ can be obtained by the tensor product of two 24-dim reps., resulting in a sym and antisym. tensor as follows,

$$\boxed{24} \otimes \boxed{24} \hat{=} \boxed{24|25} \oplus \begin{array}{|c|}\hline 24 \\ \hline 23 \\ \hline \end{array} \hat{=} 300 + 276$$

symm. rank 2 tensor

antisym. rank 2 tensor

$$= \underline{299+1} \;\; \underline{+ 276}$$

splits into traceless sym. tensor + ~~trace~~: $\square\square + \circ$

Why in lecture 324 and 300?

$N = \tilde{N} = 2$: $\alpha_{-2}^i \tilde{\alpha}_{-2}^j |0,p\rangle \rightsquigarrow \boxed{24} \otimes \boxed{24} = \square\square + \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array} = 300 + 276$

$\alpha_{-2}^i \tilde{\alpha}_{-1}^j \alpha_{-1}^k |0,p\rangle \rightsquigarrow \underset{300}{\square\square} \otimes \underset{24}{\square} = (\underset{299}{\square\square} + \underset{(1)}{\circ}) \times \underset{24}{\square}$

$\rightsquigarrow$ next page

e) For the closed bosonic string, the mass formula reads
$\alpha' M^2 = 4(N-1) = 4(\tilde{N}-1)$ with massless states being in the $SO(24)$ and massive states in the $SO(25)$ representation.

$\underline{N=\tilde{N}=0}$: $\alpha' M^2 = -4 \rightsquigarrow M^2 < 0 \rightsquigarrow$ LG $SO(25)$

$\underline{N=\tilde{N}=1}$: $\alpha' M^2 = 0 \rightsquigarrow M^2 = 0 \rightsquigarrow$ LG $SO(24)$

$\underline{N=\tilde{N}=2}$; $\alpha' M^2 = 4 \rightsquigarrow M^2 > 0 \rightsquigarrow$ LG $SO(25)$

For the number of states corresponding to their LG rep., we find:

$\underline{N=\tilde{N}=0}$: trivial rep., 1-dim. $\rightsquigarrow \underline{1}$ (1) $\underline{SO(25)}$

$\underline{N=\tilde{N}=1}$: 24x24-dim. rep., equal to before $\rightsquigarrow \underset{24}{\square} \times \underset{24}{\square}$ $\underline{SO(24)}$

$$= \square\square + \begin{smallmatrix}\square\\\square\end{smallmatrix} \mathrel{\hat{=}} 300 + 276$$

$$= \cancel{\square\square} + (1) + \begin{smallmatrix}\square\\\square\end{smallmatrix} \mathrel{\hat{=}} \underline{299 + 1 + 276}$$

$\underline{\text{sym. traceless}}$ (graviton) $\underline{\text{trace}}$ (dilaton) $\underline{\text{antisym.}}$ (B-Field)

spin 2, sym. + traceless from GR

spin 0

gen. gauge field ? Kamon field ?

why no phys. interpretation for open string? $\rightsquigarrow$ photon tachyon!

$\underline{N=\tilde{N}=2}$: in $\underline{SO(25)}$