Disclaimer

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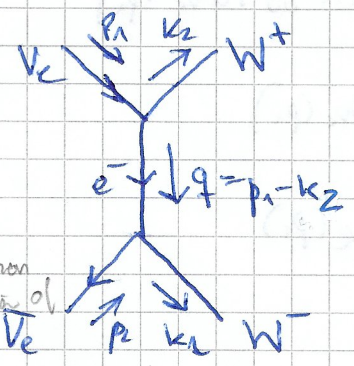
17.01.2018 Theoretical Particle Physics Exercise 13 Marvin Zanke

1) $\nu_e(p_1)\,\bar{\nu}_e(p_2) \longrightarrow W^-(k_1)\,W^+(k_2)$

with $m_e = m_{\nu_e} = 0$

What if direction of e^- defined the other way around?
⇝ squared in the end; interference, why are conventions W instead of W+ possible? → direction of fermion flow $\bar{\nu}_e$
Turn e^- direction around?
⇝ No, fermion flow and charge

a) 

$= iM$

prod. of W^+, W^- ⇝ high center of mass energy and thus neglect small masses

$$iM = \bar{v}_{\nu_e}(p_2)\frac{ig}{2\sqrt{2}}\gamma_\mu(1-\gamma_5)\frac{i(\underbrace{\not{q}}+\overbrace{m_e}^{\approx 0})}{q^2-\underbrace{m_e^2}_{\approx 0}}\frac{ig}{2\sqrt{2}}\gamma_\nu(1-\gamma_5)u_{\nu_e}(p_1)\,\epsilon^{*\mu}(k_1)\epsilon^{*\nu}(k_2)$$

longit. helicity or polarization? ⇝ same for high energy, ap in direction of k

Why still transverse to k?

b) $\epsilon_L(k) = \frac{1}{M_W}(|\vec{k}|, E\vec{e}_k)$ with $k = (E, \vec{k})$ (*)

and $\vec{e}_k = \vec{k}/|\vec{k}|$

This is equal to:

$$\epsilon_L(k) = \frac{k}{M_W} + \frac{E-|\vec{k}|}{M_W}(-1, \vec{e}_k)$$

$$= \frac{1}{M_W}(E, \vec{k}) + \frac{1}{M_W}(|\vec{k}|-E, E\vec{e}_k - \vec{k})$$

$$= \frac{1}{M_W}(|\vec{k}|, E\vec{e}_k) = (*)$$

only neglect-able compared to s? Don't know if these are terms ~s?
⇝ $q^2 \sim 2p_1k_2$ and mom. transfer much greater than M_W^2

c) Assume $s \gg M_W^2$ and thus $M_W \approx 0$ in our case

We have

$$(\not{p}_1 - \not{k}_2)\not{k}_2 u(p_1) = (\not{p}_1\not{k}_2 - \not{k}_2\not{k}_2)u(p_1)$$

$$\left|\ \not{k}\not{k} = k_\mu\gamma^\mu k_\nu\gamma^\nu = \frac{1}{2}k_\mu k_\nu(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu) = k_\mu k^\mu\right.$$

$$= (\not{p}_1\not{k}_2 - k_2^2)u(p_1) \overset{M_W\approx 0}{\approx} \not{p}_1\not{k}_2 u(p_1) = p_{1\mu}k_{2\nu}\gamma^\mu\gamma^\nu u(p_1)$$

$$= p_{1\mu}k_{2\nu}(2g^{\mu\nu} - \gamma^\nu\gamma^\mu)u(p_1) = (2p_1\cdot k_2 - \not{k}_2\not{p}_1)u(p_1)$$

$$= 2p_1k_2\,u(p_1) - \not{k}_2 m_{\nu_e} u(p_1) = 2p_1k_2\,u(p_1)$$

$$iM = \frac{-ig^2}{8} \bar{v}_{\nu_e}(p_2)\gamma_\mu(1-\gamma_5)\frac{(\not{p}_1-\not{k}_2)}{(p_1-k_2)^2}\gamma_\nu(1-\gamma_5)u_{\nu_e}(p_1)\,\epsilon^\mu(k_1)\,\epsilon^\nu(k_2)$$

eq(4) and $E=|\vec{k}|$ for $m_W\approx 0$ $\rightarrow$

$$= \frac{-ig^2}{8} \bar{v}_{\nu_e}(p_2)\gamma_\mu(1-\gamma_5)\frac{(\not{p}_1-\not{k}_2)}{(p_1-k_2)^2}\gamma_\nu(1-\gamma_5)u_{\nu_e}(p_1) \times \frac{k_1^\mu}{M_W} \times \frac{k_2^\nu}{M_W}$$

$$= \frac{-ig^2}{8M_W^2} \bar{v}_{\nu_e}(p_2)\not{k}_1(1-\gamma_5)\frac{\not{p}_1-\not{k}_2}{(p_1-k_2)^2}\not{k}_2(1-\gamma_5)u_{\nu_e}(p_1)$$

$$= \frac{-ig^2}{8M_W^2} \bar{v}_{\nu_e}(p_2)\not{k}_1(1-\gamma_5)^2\frac{\not{p}_1-\not{k}_2}{(p_1-k_2)^2}\not{k}_2\,u_{\nu_e}(p_1)$$

$$\overset{\text{Hint}}{=} \frac{-ig^2}{8M_W^2} \bar{v}_{\nu_e}(p_2)\not{k}_1(2-2\gamma_5)\frac{2p_1\cdot k_2}{-2p_1\cdot k_2}u_{\nu_e}(p_1)$$

$$= \frac{ig^2}{4M_W^2} \bar{v}_{\nu_e}(p_2)\not{k}_1(1-\gamma_5)u_{\nu_e}(p_1)$$

$$\rightsquigarrow M^+ = \frac{g^2}{4M_W^2} u^+_{\nu_e}(p_1)(1-\gamma_5^+)\gamma^0\not{k}_1\gamma^0\gamma^0\, v_{\nu_e}(p_2)$$

$$= \frac{g^2}{4M_W^2} \bar{u}_{\nu_e}(p_1)(1+\gamma_5)\not{k}_1 v_{\nu_e}(p_2)$$

$$= \frac{g^2}{4M_W^2} \bar{u}_{\nu_e}(p_1)\not{k}_1(1-\gamma_5)v_{\nu_e}(p_2)$$

$$\rightsquigarrow |M|^2 = \frac{g^4}{16M_W^4} \bar{v}_{\nu_e}(p_2)\not{k}_1(1-\gamma_5)u_{\nu_e}(p_1)\bar{u}_{\nu_e}(p_1)\not{k}_1(1-\gamma_5)v_{\nu_e}(p_2)$$

$\overline{|M|^2} = \sum_{\substack{s_1,s_2\\ u_1,u_2}} |M|^2$, where those are the spins of the particles $\nu_e, \bar{\nu}_e$

take trace cyclic
$$= \frac{g^4}{16M_W^4} \mathrm{tr}\left(\not{k}_1(1-\gamma_5)\not{p}_1\not{k}_1(1-\gamma_5)\not{p}_2\right)$$

$$= \frac{g^4}{8M_W^4} \mathrm{tr}\left(\not{k}_1(1-\gamma_5)\not{p}_1\not{k}_1\not{p}_2\right)$$

$$= \frac{g^4}{8M_W^4} \mathrm{tr}\left((1+\gamma_5)\not{k}_1\not{p}_1\not{k}_1\not{p}_2\right)$$

$$\not{k}_1\not{p}_1 = k_{1\mu}p_{1\nu}\gamma^\mu\gamma^\nu = k_{1\mu}p_{1\nu}(2g^{\mu\nu}-\gamma^\nu\gamma^\mu) = 2k_1\cdot p_1 - \not{p}_1\not{k}_1$$

$$= \frac{g^4}{8M_W^4} \mathrm{tr}\left((1+\gamma_5)(2k_1\cdot p_1 - \not{p}_1\not{k}_1)\not{k}_1\not{p}_2\right)$$

$$= \frac{g^4}{8M_W^4}\left\{\mathrm{tr}\left(2(1+\gamma_5)\not{k}_1\not{p}_2\right)k_1\cdot p_1 - \mathrm{tr}\left((1+\gamma_5)\not{p}_1\overset{\approx 0}{k_1^2}\not{p}_2\right)\right\}$$

$$= \frac{g^4}{4M_W^4}(k_1\cdot p_1)\,\mathrm{tr}\left((1+\gamma_5)\not{k}_1\not{p}_2\right)$$

What unitarity bound? $|\mathrm{Re}(a_l)| < \frac{1}{2}$? How to see from this that it shouldn't rise with s? ⇝ if it's $\sim s^{1/2}$ (or s etc) then adding up different orders and squaring them would eventually not lead to 1 ($\hat{=}$ unitarity) but grow with s

No average over neutrino spin? Sum over boson? ⇝ No average as only left-handed neutrinos energies don't matter ?

$$= \frac{g^4}{4M_W^4}(k_1\cdot p_1)\,\mathrm{tr}(\not{k}_1\not{p}_2)$$

$$= \frac{g^4}{4M_W^4}(k_1\cdot p_1)\,k_{1\mu}p_{2\nu}\,(4g^{\mu\nu})$$

$$= \frac{g^4}{M_W^4}(k_1\cdot p_1)(k_1\cdot p_2)$$

$t = (k_1-p_2)^2 = -2k_1\cdot p_2$, where $M_W^2 = 0$ was assumed

$u = (k_1-p_1)^2 = -2k_1\cdot p_1$

$s+t+u = 2M_W^2 + 2m_{\nu_e}^2 = 0$

$\Leftrightarrow s = -t-u \rightsquigarrow s^2 = t^2+u^2+2ut$

$$= \frac{g^4}{M_W^4}\left(-\frac{u}{2}\right)\left(-\frac{t}{2}\right)$$

still t and u dependence? → different way in tut. → read of from last step of iM with the $\frac{k}{}$ in d)

$$= \frac{g^4}{4M_W^4}t\cdot u = \frac{g^4}{8M_W^4}(s^2-t^2-u^2)$$

d) $\epsilon_L = \frac{k}{M_W} + \frac{E-|\vec{k}|}{M_W}(-1,\vec{e}_k) \sim k$

$\bar{\nu}\nu \propto \not{p} + m \propto s^{1/2}$

$\rightsquigarrow u \propto m^{1/2} \propto s^{1/4}$

Powers of 4-momenta : $\sqrt{s}$

Powers of spinors : $s^{1/4}$

Powers of polarization vectors : - longitudinal $\sim k \to \sqrt{s}$

- transversal : $\epsilon_+ = \frac{1}{\sqrt{2}}(0,1,i,0)$ right handed

$\epsilon_- = \frac{1}{\sqrt{2}}(0,1,-i,0)$ left handed

$\to [\epsilon_\pm] = 0$

$[M] < 0$ ok as well? ⇝ yes, suppressed are ok!

$$\Rightarrow [M] = s^{1/4}\cdot\frac{s^{1/2}}{s}s^{1/4}\sqrt{s}\sqrt{s} = s$$

$$\rightsquigarrow [M^2] = s^2$$

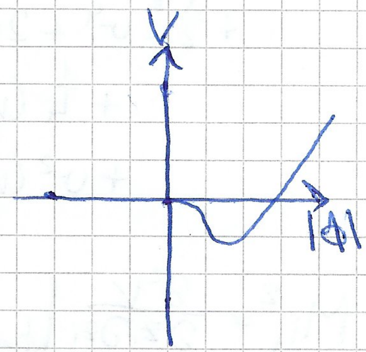
where from this power counting? → see above

$$[M_T] = s^{1/4}\frac{s^{1/2}}{s}s^{1/4} = s^0$$

$\rightsquigarrow [M^2] \sim$ const and thus no violation of parity

2) $V_{Higgs} = m^2 |\phi|^2 + \frac{\lambda}{2} |\phi|^4$

For $m^2 < 0$ (we then need $\lambda > 0$, otherwise there is no stable fixpoint), the potential looks like:



"Carrying a U(1) charge"?
⇝ inv. under U(1) leads to no other current ($e^{i\alpha}$)

Let the minimum be in the vacuum exp. value $\langle\phi\rangle = v e^{i\beta}$

$$\frac{\partial V}{\partial|\phi|} = 2m^2|\phi| + 2\lambda|\phi|^3 \overset{!}{=} 0$$

$$\overset{|\phi|\neq 0}{\leadsto} m^2 + \lambda|\phi|^2 \overset{!}{=} 0$$

$$\Leftrightarrow \langle|\phi|^2\rangle = -\frac{m^2}{\lambda} = v^2$$

Can we still derive w/ respect to ϕ? or only $|\phi|$?
⇝ w/ respect to $|\phi|$ in tutorial

Pull $()^2$ out of $\langle - \rangle$?
⇝ not always true

a)

$$\phi(x) = v e^{i\beta} + \tfrac{1}{\sqrt{2}}(\eta(x) + i\xi(x))$$
$$\phi^*(x) = v e^{-i\beta} + \tfrac{1}{\sqrt{2}}(\eta(x) - i\xi(x))$$

$$|\phi|^2 = \phi(x)\phi^*(x)$$
$$= v^2 + \tfrac{1}{\sqrt{2}} v e^{-i\beta}[\eta(x) + i\xi(x)] + \tfrac{1}{\sqrt{2}} v e^{i\beta}[\eta(x) - i\xi(x)]$$
$$+ \tfrac{1}{2}[\eta(x) + i\xi(x)][\eta(x) - i\xi(x)]$$
$$= v^2 + \tfrac{1}{\sqrt{2}} v\,\eta(x)[e^{-i\beta} + e^{i\beta}] + \tfrac{1}{\sqrt{2}} i v [e^{-i\beta} - e^{i\beta}]\xi(x)$$
$$+ \tfrac{1}{2}[\eta^2(x) + \xi^2(x)]$$
$$= v^2 + \sqrt{2}\, v\,\eta(x)\cos(\beta) + \sqrt{2}\, v \sin(\beta)\xi(x) + \tfrac{1}{2}(\eta^2(x) + \xi^2(x))$$
$$= v^2 + \sqrt{2}\, v\,(\eta(x)\cos(\beta) + \xi(x)\sin(\beta)) + \tfrac{1}{2}(\eta^2(x) + \xi^2(x))$$

$$|\phi|^4 = (|\phi|^2)^2$$
$$= v^4 + 2v^2(\eta^2(x)\cos^2(\beta) + \xi^2(x)\sin^2(\beta) + 2\eta(x)\xi(x)\cos(\beta)\sin(\beta))$$
$$+ \tfrac{1}{4}(\eta^4(x) + \xi^4(x) + 2\eta^2(x)\xi^2(x)) + 2\sqrt{2}\, v^3(\eta(x)\cos(\beta) + \xi(x)\sin(\beta))$$
$$+ 2\tfrac{v^2}{2}(\eta^2(x) + \xi^2(x)) + 2\tfrac{1}{\sqrt{2}} v(\eta(x)\cos(\beta) + \xi(x)\sin(\beta))(\eta^2(x) + \xi^2(x))$$

$$\leadsto V = m^2\left\{v^2 + \sqrt{2}\,v\,(\eta(x)\cos(\beta) + \xi(x)\sin(\beta)) + \frac{1}{2}(\eta^2(x) + \xi^2(x))\right\}$$
$$+ \frac{\lambda}{2}\Big\{v^4 + 2v^2(\eta^2(x)\cos^2(\beta) + \xi^2(x)\sin^2(\beta) + 2\eta(x)\xi(x)\cos(\beta)\sin(\beta))$$
$$+ \frac{1}{4}(\eta^4(x) + \xi^4(x) + 2\eta^2(x)\xi^2(x)) + 2\sqrt{2}\,v^3(\eta(x)\cos(\beta) + \xi(x)\sin(\beta))$$
$$+ v^2(\eta^2(x) + \xi^2(x)) + \sqrt{2}\,v\,(\eta(x)\cos(\beta) + \xi(x)\sin(\beta))(\eta^2(x) + \xi^2(x))\Big\}$$

$$M_{ij}^2 = \frac{\partial^2 V}{\partial z_i \partial z_j}\Big|_{\phi=\langle\phi\rangle}$$

$$M_{11}^2 \stackrel{!}{=} \frac{\partial}{\partial\eta}\left(m^2\eta(x) + 2v^2\lambda\eta(x)\cos^2(\beta) + \lambda v^2\eta(x)\right)\Big|_{\phi=\langle\phi\rangle}$$

only terms $\sim \eta^2(x)$ survive

$$= m^2 + 2v^2\lambda\cos^2(\beta) + \lambda v^2\ \Big|_{v^2=-m^2/\lambda}$$
$$= -2m^2\cos^2(\beta)$$

$$M_{12}^2 = M_{21}^2 = \frac{\partial}{\partial\eta}(\lambda\,\eta(x)\cos(\beta)\sin(\beta))\Big|_{\phi=\langle\phi\rangle} = 2\lambda\cos(\beta)\sin(\beta)v^2\Big|_{v^2=-m^2/\lambda}$$
$$= -2m^2\cos(\beta)\sin(\beta)$$

$$M_{22}^2 = \frac{\partial}{\partial\xi}\left(m^2\xi(x) + 2\lambda v^2\xi(x)\sin^2(\beta) + \lambda v^2\xi(x)\right)\Big|_{\phi=\langle\phi\rangle}$$
$$= m^2 + 2\lambda v^2\sin^2(\beta) + \lambda v^2\ \Big|_{v^2=-m^2/\lambda} = -2m^2\sin^2(\beta)$$

$$\leadsto M^2 = -2m^2\begin{pmatrix}\cos^2(\beta) & \cos(\beta)\sin(\beta)\\ \cos(\beta)\sin(\beta) & \sin^2(\beta)\end{pmatrix}$$

To diagonalize, we take $O = \begin{pmatrix}\cos\gamma & \sin\gamma\\ -\sin\gamma & \cos\gamma\end{pmatrix}$

and demand $M^2 O^T = O^T\begin{pmatrix}m_1^2 & 0\\ 0 & m_2^2\end{pmatrix}$, but from Goldstones theorem, we can already set $m_2^2 = 0$

$$\leadsto -2m^2\begin{pmatrix}\cos^2(\beta) & \cos(\beta)\sin(\beta)\\ \cos(\beta)\sin(\beta) & \sin^2(\beta)\end{pmatrix}\begin{pmatrix}\cos\gamma & -\sin\gamma\\ \sin\gamma & \cos\gamma\end{pmatrix} = \begin{pmatrix}\cos\gamma & -\sin\gamma\\ \sin\gamma & \cos\gamma\end{pmatrix}\begin{pmatrix}m_1^2 & 0\\ 0 & 0\end{pmatrix}$$

$$\Leftrightarrow -2m^2\begin{pmatrix}\cos^2\beta\cos\gamma + \cos\beta\sin\beta\sin\gamma & -\sin\gamma\cos^2\beta + \cos\beta\sin\beta\cos\gamma\\ \cos\beta\sin\beta\cos\gamma + \sin^2\beta\sin\gamma & -\cos\beta\sin\beta\sin\gamma + \sin^2\beta\cos\gamma\end{pmatrix} = m_1^2\begin{pmatrix}\cos\gamma & 0\\ \sin\gamma & 0\end{pmatrix}\ (*)$$

$$\leadsto \cos\beta\sin\beta\cos\gamma = \sin\gamma\cos^2\beta \quad \xrightarrow{\beta \neq \pi/2 + 2\pi} \quad \sin\beta\cos\gamma = \sin\gamma\cos\beta$$
$$\cos\beta\sin\beta\sin\gamma = \sin^2\beta\cos\gamma \quad \beta \neq 0\pi \quad \cos\beta\sin\gamma = \sin\beta\cos\gamma$$

$$\leadsto \tan\beta = \tan\gamma \leadsto \beta = \gamma$$

Why mass matrix given like this? Compare to Lagrangian → Lagrangian terms $\sim A^2$ etc (bilinear) yield mass

Do we HAVE TO insert $v = |\langle\phi\rangle|$ here already? → can already but don't have to

For $\phi = \langle\phi\rangle$ only set $\eta = \xi = 0$ in the end? → Yes, as $\langle\eta\rangle = 0$, $\langle\xi\rangle = 0$ and pert. should vanish

"global U(1) inv."?

Why 2 masses for one mass m? Broken symmetry for grand state only? Why not discussed for the other theories?

→ 2 degrees of freedom

Can't divide for first case?

$$\rightsquigarrow O = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}$$

$$(*) \Leftrightarrow -2m^2 \begin{pmatrix} \cos\beta & 0 \\ \sin\beta & 0 \end{pmatrix} = m_1^2 \begin{pmatrix} \cos\gamma \\ \sin\gamma \end{pmatrix} \quad \begin{pmatrix} \cos\gamma & 0 \\ \sin\gamma & 0 \end{pmatrix}$$

what's the other generator T^a that is not broken?

$$\rightsquigarrow m_1^2 = -2m^2 \quad \rightsquigarrow \tilde{M}^2 = \begin{pmatrix} -2m^2 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -2m^2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -2m \begin{pmatrix} x \\ y \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightsquigarrow O \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} \eta \\ 0 \end{pmatrix}$$

$$= 0 \begin{pmatrix} x \\ y \end{pmatrix} \rightsquigarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad O \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 0 \\ \xi \end{pmatrix}$$

Eigenvectors different for M^2 not diag.?

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\xrightarrow[\text{low}]{\text{energy}} SU(2) \times U(1)_{em}$$