# Disclaimer

The solution at hand was written in the course of the respective class at the University of Bonn. If not stated differently on top of the first page or the following website, the solution was prepared and handed in solely by me, Marvin Zanke. Anything in a different color than the ball pen blue is usually a correction that I or a tutor made. For more information and all my material, check:
https://www.physics-and-stuff.com/

**I raise no claim to correctness and completeness of the given solutions! This equally applies to the corrections mentioned above.**

25.10.2017 Theoretical Particle Physics Homework 3 Marvin Zanke

1) $A + B \to C + D \quad , \quad p_A + p_B = p_C + p_D$

$s = (p_A + p_B)^2 \;,\; t = (p_A - p_C)^2 \;,\; u = (p_A - p_D)^2$

a) $s + t + u = p_A^2 + p_B^2 + 2(p_A p_B) + p_A^2 + p_C^2 - 2(p_A p_C) + p_A^2 + p_D^2 - 2(p_A p_D)$

$= 3m_A^2 + m_B^2 + m_C^2 + m_D^2 + 2 p_A \cdot (p_B - p_C - p_D)$

mom. conserv.

$\overset{\downarrow}{=} 3m_A^2 + m_B^2 + m_C^2 + m_D^2 - 2p_A \cdot p_A = m_A^2 + m_B^2 + m_C^2 + m_D^2$

b) $p_A = \begin{pmatrix} E_A \\ \vec{p}_A \end{pmatrix} , p_B = \begin{pmatrix} E_B \\ \vec{p}_B \end{pmatrix} , p_C = \begin{pmatrix} E_C \\ \vec{p}_C \end{pmatrix} , p_D = \begin{pmatrix} E_D \\ \vec{p}_D \end{pmatrix}$

✓

No t,u? ⤳ possible? Alternative: $E_C = \frac{1}{2\sqrt{s}}(m_A^2 + m_B^2 - t - u + 2m_C^2)$

CMS: $\vec{p}_B = -\vec{p}_A$ & $\vec{p}_D = -\vec{p}_C$

$\sqrt{s} = \sqrt{(p_A + p_B)^2} = E_A + E_B = \sqrt{(p_C + p_D)^2} = E_C + E_D$

$\rightsquigarrow \sqrt{s} = \sqrt{|\vec{p}_{A/C}|^2 + m_{A/C}^2} + \sqrt{|\vec{p}_{B/D}|^2 + m_{B/D}^2}$

$\vec{p}_B = -\vec{p}_A$, $\vec{p}_D = -\vec{p}_C$

$\Leftrightarrow \sqrt{s} - \sqrt{|\vec{p}_{A/C}|^2 + m_{B/D}^2} = \sqrt{|\vec{p}_{A/C}|^2 + m_{A/C}^2}$

$\Leftrightarrow s + (|\vec{p}_{A/C}|^2 + m_{B/D}^2) - 2\sqrt{s}\sqrt{|\vec{p}_{A/C}|^2 + m_{B/D}^2} = |\vec{p}_{A/C}|^2 + m_{A/C}^2$

$\Leftrightarrow s + m_{B/D}^2 - m_{A/C}^2 = 2\sqrt{s}\sqrt{|\vec{p}_{A/C}|^2 + m_{B/D}^2}$

$\Leftrightarrow |\vec{p}_{A/C}|^2 = \frac{(s + m_{B/D}^2 - m_{A/C}^2)^2}{4s} - m_{B/D}^2$

$\Leftrightarrow |\vec{p}_{A/C}|^2 = \frac{s^2 + m_{B/D}^4 + m_{A/C}^4 - 2s m_{B/D}^2 - 2s m_{A/C}^2 - 2m_{B/D}^2 m_{A/C}^2}{4s}$

$= \frac{1}{4s} \lambda(s, m_{B/D}^2, m_{A/C}^2)$

with $\lambda(x,y,z) := x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$

$\rightsquigarrow E_C = \sqrt{s} - E_D = \sqrt{s} - \sqrt{m_D^2 + |\vec{p}_D|^2} = \sqrt{s} - \sqrt{m_D^2 + |\vec{p}_C|^2}$

$= \sqrt{s} - \sqrt{m_D^2 + \frac{1}{4s}(s^2 + m_D^4 + m_C^4 - 2sm_D^2 - 2sm_C^2 - 2m_D^2 m_C^2)}$

$= \sqrt{s} - \sqrt{\frac{1}{4s}(s^2 + m_D^4 + m_C^4 + 2sm_D^2 - 2sm_C^2 - 2m_D^2 m_C^2)}$

$$= \sqrt{s} - \frac{1}{2\sqrt{s}}\sqrt{(s+m_D^2-m_C^2)^2} = \sqrt{s} - \frac{s+m_D^2-m_C^2}{2\sqrt{s}}$$

$$= \frac{s-m_D^2+m_C^2}{2\sqrt{s}} = \frac{1}{2\sqrt{s}}\left(s+m_C^2-m_D^2\right)$$

c) $p_A = \begin{pmatrix} E_A \\ \vec{p}_A \end{pmatrix}$, $p_B = \begin{pmatrix} E_B \\ \vec{0} \end{pmatrix} = \begin{pmatrix} m_B \\ \vec{0} \end{pmatrix}$

$$\rightsquigarrow s = (p_A+p_B)^2 = p_A^2 + p_B^2 + 2p_A p_B = m_A^2 + m_B^2 + 2E_A m_B$$

Also possible that $s = (E_A + m_B)^2 - |\vec{p}_A|^2$ $\rightsquigarrow$ same in tutorial ($E_A^2 = p_A^2 + m_A^2$ yields the same)

d) $t = (p_A - p_C)^2 = m_A^2 + m_C^2 - 2p_A p_C = m_A^2 + m_C^2 - 2E_A E_C + 2\vec{p}_A \cdot \vec{p}_C$

$$= m_A^2 + m_C^2 - 2\sqrt{|\vec{p}_A|^2 + m_A^2}\sqrt{|\vec{p}_C|^2 + m_C^2} + 2|\vec{p}_A||\vec{p}_C|\cos\theta$$

$|\vec{p}_C|$ fixed

$\rightsquigarrow$ with b) $m_A^2 + m_C^2 - 2\sqrt{|\vec{p}_A|^2 + m_A^2}\sqrt{|\vec{p}_C|^2 + m_C^2} - 2|\vec{p}_A||\vec{p}_C| \leq t$

$$t \leq m_A^2 + m_C^2 - 2\sqrt{|\vec{p}_A|^2 + m_A^2}\sqrt{|\vec{p}_C|^2 + m_C^2} + 2|\vec{p}_A||\vec{p}_C|$$

Analogously for $u$ w/ $\vec{p}_C \mapsto \vec{p}_D$, $m_C \mapsto m_D$

2) $d\sigma = \frac{1}{4E_1E_2|\vec{v}|}(2\pi)^4\,\delta^{(4)}(p_3+p_4-p_1-p_2)\,\frac{d^3p_3}{2E_3(2\pi)^3}\,\frac{d^3p_4}{2E_4(2\pi)^3}\,|F|^2$

for $1+2 \longrightarrow 3+4$

a) $4E_1E_2|\vec{v}| \doteq 4\left[(p_1\cdot p_2)^2 - m_1^2m_2^2\right]^{1/2} \doteq 2\lambda^{1/2}(s,m_1^2,m_2^2)$

for the incident flux factor in CMS and Lab frame.

We first prove the $2^{nd}$ equality for an arbitrary frame:

We will use $s=(p_1+p_2)^2 = m_1^2+m_2^2+2p_1p_2$

$\Leftrightarrow \quad p_1\cdot p_2 = \frac{1}{2}(s-m_1^2-m_2^2)$

$$4\left[(p_1\cdot p_2)^2-m_1^2m_2^2\right]^{1/2} = 4\left[\frac{1}{4}(s-m_1^2-m_2^2)^2-m_1^2m_2^2\right]^{1/2}$$

$$= 2\left[(s^2+m_1^4+m_2^4-2sm_1^2-2sm_2^2+2m_1^2m_2^2)-4m_1^2m_2^2\right]^{1/2}$$

$$= 2\left[s^2+m_1^4+m_2^4-2sm_1^2-2sm_2^2-2m_1^2m_2^2\right]^{1/2}$$

$$= 2\lambda^{1/2}(s,m_1^2,m_2^2)$$

Now, we will prove the first stated equality for a) rest frame of particle 2 and b) the CMS-frame.

a) $p_1 = \begin{pmatrix} E_1 \\ \vec{p}_1 \end{pmatrix},\ p_2 = \begin{pmatrix} m_2 \\ \vec{0} \end{pmatrix},\ |\vec{v}| = |\vec{v}_1| = \left|\frac{\vec{p}_1}{E_1}\right|$, as $p=\gamma m v = Ev$

$$4E_1E_2|\vec{v}| = 4E_1E_2\left|\frac{\vec{p}_1}{E_1}\right| = 4E_2|\vec{p}_1| = 4E_2\sqrt{E_1^2-m_1^2}$$

$$= 4\sqrt{E_1^2E_2^2-E_2^2m_1^2} \doteq 4\sqrt{(E_1E_2)^2-m_1^2m_2^2}$$

$$= 4\sqrt{(E_1m_2)^2-m_1^2m_2^2} = 4\sqrt{(p_1\cdot p_2)^2-m_1^2m_2^2}$$

b) $p_1 = \begin{pmatrix} E_1 \\ \vec{p}_1 \end{pmatrix},\ p_2 = \begin{pmatrix} E_2 \\ -\vec{p}_1 \end{pmatrix}$, $p=\gamma m v = Ev$

$$4E_1E_2|\vec{v}| \overset{\vec{v}=\vec{v}_1-\vec{v}_2}{=} 4E_1E_2\left|\frac{\vec{p}_1}{E_1} - \frac{-\vec{p}_1}{E_2}\right| = 4E_1E_2|\vec{p}_1|\left|\frac{E_1+E_2}{E_1E_2}\right| = 4|\vec{p}_1|\sqrt{s}$$

$$\overset{1b)}{=} 4\left(\frac{1}{2\sqrt{s}}\lambda^{1/2}(s,m_1^2,m_2^2)\right)\sqrt{s} = 2\lambda^{1/2}(s,m_1^2,m_2^2) \quad \square$$

Do we really have to perform the phase space integrals?

b) $$d\sigma = \frac{1}{4E_1E_2|\vec{v}|}(2\pi)^4\delta^{(4)}(p_3+p_4-p_1-p_2)\frac{d^3p_3}{2E_3(2\pi)^3}\frac{d^3p_4}{2E_4(2\pi)^3}|F|^2$$

$$\sigma = \int\frac{d^3p_3\,d^3p_4}{4E_3E_4(2\pi)^2}\frac{1}{2\lambda^{1/2}(s,m_1^2,m_2^2)}\delta^{(4)}(p_3+p_4-p_1-p_2)|F|^2$$

$$= \int\frac{d^3p_3}{16\pi^2E_3E_4}\frac{1}{2\lambda^{1/2}(s,m_1^2,m_2^2)}\delta(E_3+E_4-E_1-E_2)|F|^2$$

What for the stars at $|\vec{p}^*|$? → CMS-frame

$d^3p_3 \equiv d^3p^* = |\vec{p}^*|^2 d|\vec{p}^*| d\Omega^*$

$$= \int d\Omega^* d|\vec{p}^*|\,|\vec{p}^*|^2\frac{1}{32\pi^2E_3E_4\lambda^{1/2}(s,m_1^2,m_2^2)}\delta(E_1+E_2\underbrace{-E_3-E_4}_{-\sqrt{s}})|F|^2$$

$\sqrt{s} = E_3+E_4$ or $= E_1+E_2$? Already used the $\delta$-distribution?

$$\sqrt{s} = E_3+E_4 = \sqrt{|\vec{p}^*|^2+m_3^2}+\sqrt{|\vec{p}^*|^2+m_4^2}$$

$$\rightsquigarrow \frac{d\sqrt{s}}{d|\vec{p}^*|} = \frac{|\vec{p}^*|}{E_3}+\frac{|\vec{p}^*|}{E_4} = |\vec{p}^*|\frac{E_3+E_4}{E_3E_4}$$

$$\rightsquigarrow d|\vec{p}^*| = d\sqrt{s}\,\frac{E_3E_4}{|\vec{p}^*|(E_3+E_4)} = d\sqrt{s}\,\frac{E_3E_4}{|\vec{p}^*|\sqrt{s}}$$

Now again set $E_3+E_4=\sqrt{s}$ after integration?

$$= \int d\Omega^* d\sqrt{s}\,|\vec{p}^*|\frac{1}{E_3+E_4}\frac{1}{32\pi^2\lambda^{1/2}(s,m_1^2,m_2^2)}\delta(\sqrt{s}-E_1-E_2)|F|^2$$

$$= \int d\Omega^*\,|\vec{p}^*|\frac{1}{32\pi^2\sqrt{s}\,\lambda^{1/2}(s,m_1^2,m_2^2)}|F|^2, \quad \sqrt{s}=E_1+E_2$$

What if I wanted $s=(E_1+E_2)^2=(E_3+E_4)^2$ $\rightsquigarrow \frac{ds}{d|\vec{p}|}$ etc but $\delta$-distribution now w/ $\delta(f(x)) = \frac{1}{f'(x)}\delta(x-x_0)$

1b) $|\vec{p}^*|^2 = \frac{\lambda(s,m_3^2,m_4^2)}{4s}$

$$= \int d\Omega^*\frac{1}{64\pi^2s\,\lambda^{1/2}(s,m_1^2,m_2^2)}\lambda^{1/2}(s,m_3^2,m_4^2)|F|^2$$

$$\rightsquigarrow \frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2s\,\lambda^{1/2}(s,m_1^2,m_2^2)}\lambda^{1/2}(s,m_3^2,m_4^2)|F|^2$$

Just drop the integral $\int d\Omega^*$ now again? → shouldn't have integrated about it from the beginning...

c) $$t = (p_1-p_3)^2 = m_1^2+m_3^2-2p_1p_3 = m_1^2+m_3^2-2(E_1E_3-2|\vec{p}_1||\vec{p}_3|\cos\theta^*)$$

$$d\Omega^* = d\varphi^*\,d\cos\theta^*$$

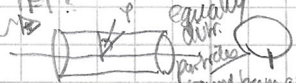
$$\rightsquigarrow \frac{dt}{d\cos\theta^*} = 2|\vec{p}_1||\vec{p}_3| \Leftrightarrow d\cos\theta^* = dt\,\frac{1}{2|\vec{p}_1||\vec{p}_3|}$$

$$\Rightarrow \frac{d\sigma}{d\Omega^*} = \frac{d\sigma}{d\varphi^*\,d\cos\theta^*} = \frac{1}{64\pi^2s\,\lambda^{1/2}(s,m_1^2,m_2^2)}\lambda^{1/2}(s,m_3^2,m_4^2)|F|^2$$

Cylindrical sym. about axis for what? $|F|^2$?

$$\rightsquigarrow \frac{d\sigma}{d\cos\theta^*} = \frac{1}{32\pi s\,\lambda^{1/2}(s,m_1^2,m_2^2)}\lambda^{1/2}(s,m_3^2,m_4^2)|F|^2$$

$$= \frac{d\sigma}{dt}\,2|\vec{p}_1||\vec{p}_3|$$



$$\rightsquigarrow \frac{d\sigma}{dt} = \frac{1}{32\pi s \lambda^{1/2}(s,m_1^2,m_2^2)} \lambda^{1/2}(s,m_3^2,m_4^2) |F|^2 \frac{1}{2|\vec{p}_1||\vec{p}_3|}$$

$|\vec{p}_1|^2 = \frac{\lambda(s,m_1^2,m_2^2)}{4s}$ (1b)

$|\vec{p}_3|^2 = \frac{\lambda(s,m_3^2,m_4^2)}{4s}$

$$\Rightarrow \frac{1}{64\pi s \lambda^{1/2}(s,m_1^2,m_2^2)} \lambda^{1/2}(s,m_3^2,m_4^2) |F|^2 \cdot \frac{4s}{\lambda^{1/2}(s,m_1^2,m_2^2)\lambda^{1/2}(s,m_3^2,m_4^2)}$$

$$= \frac{1}{16\pi \lambda(s,m_1^2,m_2^2)} |F|^2$$

What for $\frac{d\sigma}{dt}$?

$\rightsquigarrow$ dependence of cross section w/ mom. transfer (big momentum transfer ~ in experiment) (large momentum transfer) as cylindrical symmetry $\Rightarrow d\sigma/d\Omega$ not really much information

3). • $\gamma_\mu \gamma^\mu = \frac{1}{2}\gamma_\mu\gamma^\mu + \frac{1}{2}\gamma^\mu\gamma_\mu = \frac12 g_{\mu\kappa}\gamma^\kappa\gamma^\mu + \frac12 g_{\mu\kappa}\gamma^\mu\gamma^\kappa$

$= \frac12 g_{\mu\kappa}\{\gamma^\kappa,\gamma^\mu\} = g_{\mu\kappa} g^{\kappa\mu}\mathbb{1} = 4\cdot\mathbb{1}$

• $\gamma_\mu \not a \gamma^\mu = \gamma_\mu a_\kappa \gamma^\kappa\gamma^\mu = a_\kappa\gamma_\mu(-\gamma^\mu\gamma^\kappa + \{\gamma^\kappa,\gamma^\mu\}) = -a_\kappa(4\mathbb{1})\gamma^\kappa + 2a_\kappa\gamma_\mu g^{\kappa\mu}$

$= -4\not a + 2\not a = -2\not a$

• $\gamma_\mu\not a\not b\gamma^\mu = \gamma_\mu a_\kappa\gamma^\kappa b_\lambda\gamma^\lambda\gamma^\mu = a_\kappa b_\lambda \gamma_\mu\gamma^\kappa\gamma^\lambda\gamma^\mu = a_\kappa b_\lambda\gamma_\mu\gamma^\kappa(-\gamma^\mu\gamma^\lambda + \{\gamma^\lambda,\gamma^\mu\})$

$= -a_\kappa b_\lambda\gamma_\mu\gamma^\kappa\gamma^\mu\gamma^\lambda + 2a_\kappa b_\lambda\gamma_\mu\gamma^\kappa g^{\lambda\mu}$

$\overset{\text{"b)"}}{=} -b_\lambda(-2\not a)\gamma^\lambda + 2\not b\not a = 2\not a\not b + 2\not b\not a$

$= 2a_\kappa\gamma^\kappa b_\lambda\gamma^\lambda + 2\not b\not a = 2a_\kappa b_\lambda(-\gamma^\lambda\gamma^\kappa + \{\gamma^\kappa,\gamma^\lambda\}) + 2\not b\not a$

$= -2\not b\not a + 4a_\kappa b_\lambda g^{\kappa\lambda} + 2\not b\not a = 4(a\cdot b)$

• $\gamma_\mu\not a\not b\not c\gamma^\mu = \gamma_\mu a_\kappa\gamma^\kappa b_\lambda\gamma^\lambda c_\eta\gamma^\eta\gamma^\mu = a_\kappa b_\lambda c_\eta\gamma_\mu\gamma^\kappa\gamma^\lambda\gamma^\eta\gamma^\mu$

$= a_\kappa b_\lambda c_\eta\gamma_\mu\gamma^\kappa\gamma^\lambda(-\gamma^\mu\gamma^\eta + \{\gamma^\eta,\gamma^\mu\})$

$= -a_\kappa b_\lambda c_\eta\gamma_\mu\gamma^\kappa\gamma^\lambda\gamma^\mu\gamma^\eta + 2a_\kappa b_\lambda c_\eta\gamma_\mu\gamma^\kappa\gamma^\lambda g^{\eta\mu}$

$\overset{\text{"c)"}}{=} -c_\eta(4(a\cdot b))\gamma^\eta + 2\not c\not a\not b = -4(a\cdot b)\not c + 2\not c\not a\not b$

$= -4(a\cdot b)\not c + 2c_\kappa\gamma^\kappa a_\lambda\gamma^\lambda b_\eta\gamma^\eta$

$= -4(a\cdot b)\not c + 2c_\kappa a_\lambda b_\eta\gamma^\kappa(-\gamma^\eta\gamma^\lambda + \{\gamma^\lambda,\gamma^\eta\})$

$= -4(a\cdot b)\not c - 2\not c\not b\not a + 4c_\kappa a_\lambda b_\eta\gamma^\kappa g^{\lambda\eta}$

$= -4(a\cdot b)\not c - 2\not c\not b\not a + 4(a\cdot b)\not c = -2\not c\not b\not a$

what for $\gamma_5$ explicitly given?

• $\mathrm{Tr}(\underbrace{\gamma^\mu\gamma^\nu\ldots\gamma^\lambda}_{\text{odd \#}}) \underset{\uparrow (\gamma_5)^2 = \mathbb{1}}{=} \mathrm{Tr}((\gamma_5)^2\gamma^\mu\gamma^\nu\ldots\gamma^\lambda) \underset{\text{cyclic}}{=} \mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\ldots\gamma^\lambda\gamma_5)$

$\{\gamma_5,\gamma^\mu\} = 0 \Rightarrow\ = \mathrm{Tr}((-1)^{\#\gamma's}(\gamma_5)^2\gamma^\mu\gamma^\nu\ldots\gamma^\lambda) = (-1)\,\mathrm{Tr}(\gamma^\mu\gamma^\nu\ldots\gamma^\lambda)$

$\Rightarrow \mathrm{Tr}(\gamma^\mu \ldots \gamma^\lambda) =$

- $\mathrm{Tr}(\not{a}\not{b}) = \mathrm{Tr}(a_\kappa \gamma^\kappa b_\lambda \gamma^\lambda) = a_\kappa b_\lambda \mathrm{Tr}(\gamma^\kappa \gamma^\lambda) = a_\kappa b_\lambda \mathrm{Tr}(-\gamma^\lambda\gamma^\kappa + \{\gamma^\kappa, \gamma^\lambda\})$

$$= -a_\kappa b_\lambda \mathrm{Tr}(\gamma^\lambda \gamma^\kappa) + 2 a_\kappa b_\lambda \mathrm{Tr}(g^{\kappa\lambda})$$

$$= -\mathrm{Tr}(\not{b}\not{a}) + 8(a\cdot b)$$

$$\Longleftrightarrow \mathrm{Tr}(\not{a}\not{b} + \not{b}\not{a}) = 8(a\cdot b) \underset{\uparrow \text{cyclic}}{=} 2\mathrm{Tr}(\not{a}\not{b})$$

$$\Longleftrightarrow \mathrm{Tr}(\not{a}\not{b}) = 4(a\cdot b)$$

- $\mathrm{Tr}(\not{a}\not{b}\not{c}\not{d}) = \mathrm{Tr}(a_\kappa \gamma^\kappa b_\lambda \gamma^\lambda c_\eta \gamma^\eta d_\omega \gamma^\omega) = a_\kappa b_\lambda c_\eta d_\omega \mathrm{Tr}(\gamma^\kappa \gamma^\lambda \gamma^\eta \gamma^\omega)$

$$= a_\kappa b_\lambda c_\eta d_\omega \mathrm{Tr}([-\gamma^\lambda \gamma^\kappa + \{\gamma^\kappa, \gamma^\lambda\}]\gamma^\eta \gamma^\omega)$$

$$= -a_\kappa b_\lambda c_\eta d_\omega \mathrm{Tr}(\gamma^\lambda \gamma^\kappa \gamma^\eta \gamma^\omega) + 2 g^{\kappa\lambda} a_\kappa b_\lambda c_\eta d_\omega \mathrm{Tr}(\gamma^\eta \gamma^\omega)$$

$$\overset{\text{"f1"}}{=} -a_\kappa b_\lambda c_\eta d_\omega \mathrm{Tr}(\gamma^\lambda [-\gamma^\eta \gamma^\kappa + \{\gamma^\kappa, \gamma^\eta\}]\gamma^\omega) + 2(a\cdot b)\cdot 4(c\cdot d)$$

$$= a_\kappa b_\lambda c_\eta d_\omega \mathrm{Tr}(\gamma^\lambda \gamma^\eta \gamma^\kappa \gamma^\omega) - 2 g^{\kappa\eta} a_\kappa b_\lambda c_\eta d_\omega \mathrm{Tr}(\gamma^\lambda \gamma^\omega) + 8(a\cdot b)(c\cdot d)$$

$$\overset{\text{"f1"}}{=} a_\kappa b_\lambda c_\eta d_\omega \mathrm{Tr}(\gamma^\lambda \gamma^\eta [-\gamma^\omega \gamma^\kappa + \{\gamma^\kappa, \gamma^\omega\}]) - 2(a\cdot c)\cdot 4(b\cdot d) + 8(a\cdot b)(c\cdot d)$$

$$= -a_\kappa b_\lambda c_\eta d_\omega \mathrm{Tr}(\gamma^\lambda \gamma^\eta \gamma^\omega \gamma^\kappa) + 2 g^{\kappa\omega} a_\kappa b_\lambda c_\eta d_\omega \mathrm{Tr}(\gamma^\lambda \gamma^\eta)$$
$$- 8(a\cdot c)(b\cdot d) + 8(a\cdot b)(c\cdot d)$$

$$= -\mathrm{Tr}(\not{b}\not{c}\not{d}\not{a}) + 8(a\cdot d)(b\cdot c) - 8(a\cdot c)(b\cdot d) + 8(a\cdot b)(c\cdot d)$$

$$\Longleftrightarrow \mathrm{Tr}(\not{a}\not{b}\not{c}\not{d} + \not{b}\not{c}\not{d}\not{a}) = 8\{(a\cdot b)(c\cdot d) + (a\cdot d)(b\cdot c) - (a\cdot c)(b\cdot d)\}$$

$$\underset{\text{cyclic}}{\overset{\text{Trace}}{\Longleftrightarrow}} \mathrm{Tr}(\not{a}\not{b}\not{c}\not{d}) = 4\{(a\cdot b)(c\cdot d) + (a\cdot d)(b\cdot c) - (a\cdot c)(b\cdot d)\}$$