Disclaimer

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https://www.physics-and-stuff.com/

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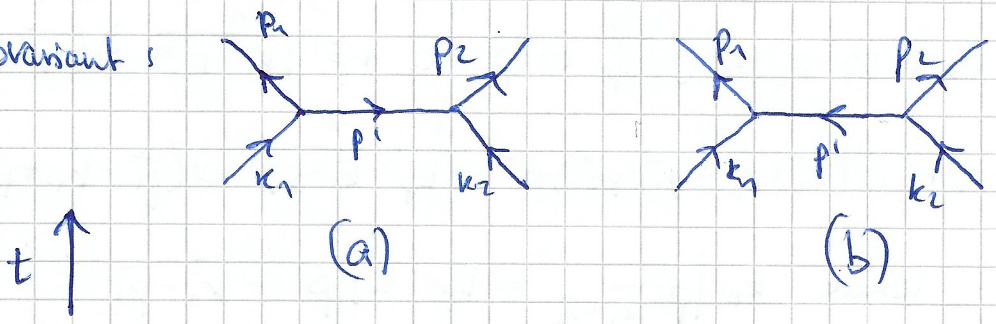
09.11.2017 **Theoretical Particle Physics Exercise 5** Marvin Zanke

1) a) $\phi_{k_1} + \phi_{k_2} \longrightarrow \phi_{p_1} + \phi_{p_2}$, t-channel !

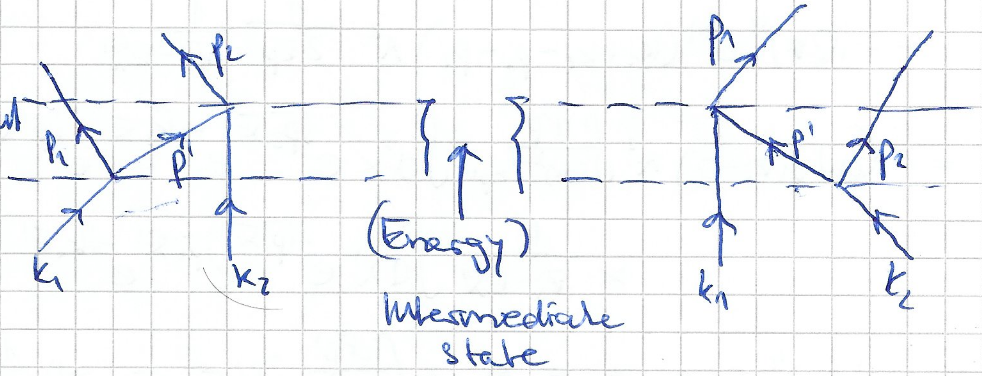
✓

Why do we work in the Schrödinger picture?
~> no time dep in fields; $\phi(\vec{x})$

Covariant :



Non-covariant



✓

Why „on shell" and „own energy"?
~> particles described by ϕ's are on shell, V_{ka} isn't
That's why / 2nd order

$$\rightsquigarrow \text{(a)} : E_n^{(a)} = E_{p_1} + E_{k_2} + E_{p'}$$

$$\text{(b)} : E_n^{(b)} = E_{p_2} + E_{k_1} + E_{p'}$$

$E_{p'}$ fixed energy?
~> we don't know the energy
~> energy cons. at vertex ✓

b) this means that the energy isn't conserved for the intermediate state / at the vertices. It can have arbitrarily high energies / momenta. One can see this, by evaluating the integrals to delta distributions, yielding $\delta^{(3)}$ instead of $\delta^{(4)}$.

But have δ -> for energy conservation?
~> total energy cons.

c) $$A_{fi}^{(2)} = -i(2\pi)\delta(E_i - E_f)\sum_{n\neq i}\left\{\int d^3x\, \phi_f^*(\vec{x})V_{ka}(\vec{x})\phi_n(\vec{x})\right\}\frac{1}{E_i - E_n}$$
$$\times\left\{\int d^3x'\, \phi_n^*(\vec{x}')V_{ka}(\vec{x}')\,\phi_i(\vec{x}')\right\}$$

Why replace sum by integral?

✓

$V_{ka}(\vec{x})$ and $V_{ka}(\vec{x}')$ same momenta?
~> Same particle; ϕ

$$\xrightarrow[\text{int.}]{\text{continuum of states}} -i(2\pi)\delta(E_i - E_f)\int\frac{d^3p'}{(2\pi)^3 2E_{p'}}\left\{\int d^3x\, \phi_f^*(\vec{x})V_{ka}(\vec{x})\phi_n(\vec{x})\right\}$$
$$\times\frac{1}{E_i - E_n}\left\{\int d^3x'\, \phi_n^*(\vec{x}')V_{ka}(\vec{x}')\,\phi_i(\vec{x}')\right\}$$

$$A_{fi}^{(a)} = -i(2\pi)\delta(E_i-E_f)A^2\int\frac{d^3p'}{(2\pi)^3 2E_{p'}}\int d^3x\, e^{\underset{p_2}{ip_1\vec{x}}}\, e^{\underset{-k_2}{-ik_1\vec{x}}}\, e^{\underset{-p'}{i\vec{p}'\vec{x}}}$$

Change! Which particle ϕ_i / ϕ_f^* ! 2 outg. / incom.? ⇝ last one out = outg

↕ change order

$$\times\frac{1}{E_i-E_n^{(a)}}\int d^3x'\, e^{\underset{p_n}{-i\vec{p}'\vec{x}'}}\, e^{\underset{p_1}{i\vec{p}_2\vec{x}'}}\, e^{\underset{-k_1}{-i\vec{k}_2\vec{x}'}}$$

$\phi_f^* = p_2$ is outgoing

$\phi_i = \phi_{k_1}$

ϕ_n^* for outgoing from considered vertex

$$= -i(2\pi)^4\delta(E_i-E_f)A^2\int\frac{d^3p'}{2E_{p'}}\delta(\vec{p}_1-\vec{k}_1+\vec{p}')\frac{1}{E_i-E_n^{(a)}}\delta(-\vec{p}'+\vec{p}_2-\vec{k}_2)$$

$$= -i(2\pi)^4\delta(k_1^0+k_2^0-p_1^0-p_2^0)A^2\frac{1}{2E_{p'}}\frac{1}{E_i-E_n^{(a)}}\delta(\vec{p}_1-\vec{k}_1+\vec{p}_2-\vec{k}_2)$$

Why only $e^{i\vec{k}\vec{x}}$? What about time evolution ⇝ not included here; is hidden in the fact, that also ϕ_f^* is different in both time-ordered diagrams

where $E_{p'}\big|_{\vec{p}'=\vec{k}_1-\vec{p}_1} = E_{p'}$ w/ $\vec{p}' := \vec{k}_1-\vec{p}_1$

$$= -i(2\pi)^4\delta^{(4)}(k_1+k_2-p_1-p_2)A^2\frac{1}{2E_{p'}}\frac{1}{E_i-E_n^{(a)}}$$

$$A_{fi}^{(b)} = -i(2\pi)\delta(E_i-E_f)A^2\int\frac{d^3p'}{(2\pi)^3 2E_{p'}}\int d^3x\, e^{i\vec{p}_2\vec{x}}\, e^{-i\vec{k}_2\vec{x}}\, e^{i\vec{p}'\vec{x}}$$

$$\times\frac{1}{E_i-E_n^{(b)}}\int d^3x'\, e^{-i\vec{p}'\vec{x}'}\, e^{i\vec{p}_1\vec{x}'}\, e^{-i\vec{k}_1\vec{x}'}$$

$$= -i(2\pi)^4\delta(k_1^0+k_2^0-p_1^0-p_2^0)A^2\int\frac{d^3p'}{2E_{p'}}\delta(\vec{p}_2-\vec{k}_2+\vec{p}')\frac{1}{E_i-E_n^{(b)}}\delta(\vec{p}'+\vec{p}_1-\vec{k}_1)$$

$$= -i(2\pi)^4\delta(k_1+k_2-p_1-p_2)A^2\frac{1}{2E_{p'}}\frac{1}{E_i-E_n^{(b)}}$$ w/ $E_{p'}$ defined as in (a)

Why $E_n \neq E_{p'}$

$$\rightsquigarrow A_{fi}^{(a)}+A_{fi}^{(b)} = -i(2\pi)^4\delta(k_1+k_2-p_1-p_2)A^2\frac{1}{2E_{p'}}\left\{\underbrace{\frac{1}{E_i-E_n^{(a)}}+\frac{1}{E_i-E_n^{(b)}}}_{(*)}\right\}$$

$$(*) \overset{a)}{=} \frac{1}{E_i-E_{p_1}-E_{k_2}-E_{p'}}+\frac{1}{E_i-E_{p_2}-E_{k_1}-E_{p'}}$$, w/ $E_i = E_{k_1}+E_{k_2}$

$E_i = E_{k_1}+E_{k_2}$

$$= \frac{1}{E_{k_1}-E_{p_1}-E_{p'}}+\frac{1}{E_{k_2}-E_{p_2}-E_{p'}}$$

$$\overset{E_{k_1}+E_{k_2}=E_{p_1}+E_{p_2}}{=} \frac{1}{E_{p_1}-E_{k_1}-E_{p'}}+\frac{1}{E_{k_1}-E_{p_1}-E_{p'}} = \frac{1}{x-y}+\frac{1}{-x-y} = \frac{1}{x-y}-\frac{1}{x+y}$$

$$= \frac{x+y-x+y}{(x+y)(x-y)} = \frac{2y}{x^2-y^2} = \frac{2E_{p'}}{(E_{k_1}-E_{p_1})^2-E_{p'}^2}$$

$$= -i(2\pi)^4\delta(k_1+k_2-p_1-p_2)A^2\frac{1}{(E_{k_1}-E_{p_1})^2-E_{p'}^2}$$

Why E_i' energy of outgoing particle always E_{p_1}?

d)

$$\text{CMS}: \; k_1 = \begin{pmatrix} E_{k_1} \\ \vec{k} \end{pmatrix}, \; k_2 = \begin{pmatrix} E_{k_2} \\ -\vec{k} \end{pmatrix}, \; p_1 = \begin{pmatrix} E_{p_1} \\ \vec{p} \end{pmatrix}, \; p_2 = \begin{pmatrix} E_{p_2} \\ -\vec{p} \end{pmatrix}$$

✓

In CMS $\begin{pmatrix} \\ -\vec{k} \end{pmatrix}$ $\begin{pmatrix} \\ \vec{k} \end{pmatrix}$ possible for t-channel? ⇝ always possible, also considering $s=(k_1+k_2)^2$

$$s = (k_1+k_2)^2 = (p_1+p_2)^2 = (E_{k_1}+E_{k_2})^2 = (E_{p_1}+E_{p_2})^2$$

and

$$E_{k_1} = \sqrt{\vec{k}_1^2+m^2} = \sqrt{\vec{k}^2+m^2} = E_{k_2} \equiv E_k$$

$$E_{p_1} = \sqrt{\vec{p}_1^2+m^2} = \sqrt{\vec{p}^2+m^2} = E_{p_2} \equiv E_p$$

$$\rightsquigarrow \; s = (E_k)^2 = 4E_k^2 \qquad \rightsquigarrow E_k = E_p = \frac{\sqrt{s}}{2}$$

$$s = (2E_p)^2 = 4E_p^2$$

✓

In this frame $E_1 = E_1'$? ⇝ in CMS $E_k = E_p$

$$\rightsquigarrow E_{k_1} = E_{p_1} \text{ and thus}$$

✓

Completes the rederivation of the result? ⇝ $E_{k_1} = E_{p_1}$ ⇝ prop. given by $\frac{1}{t-m_\phi^2}$

$$t = (k_1-p_1)^2 = (E_{k_1}-E_{p_1})^2 - (\vec{k}-\vec{p})^2 = -(\vec{k}-\vec{p})^2 = -\vec{p}'^2$$

$$\rightsquigarrow E_{p'}^2 = \vec{p}'^2 + m^2 = -t + m^2$$

2) $$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \sum_i \overline{\psi}_i \underbrace{(i\cancel{\partial} - m_i - q_i \cancel{A})}_{= i\cancel{D}_i - m_i \text{, w/ } D_{\mu_i} = \partial_\mu + iq_i A_\mu} \psi_i$$

Invariant under $\psi_i \mapsto e^{i\alpha q_i}\psi_i$, global transformation

⇝ Noether current: $J_Q^\mu = -J_N^\mu = \sum_i q_i \overline{\psi}_i \gamma^\mu \psi_i$

a) E.L. eq.: $$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \overline{\psi}_j)} - \frac{\partial\mathcal{L}}{\partial\overline{\psi}_j} = 0$$

$$⇝ \partial_\mu \cdot 0 - (i\cancel{\partial} - m_i - q_i\cancel{A})\psi_j = 0$$

$$\Longleftrightarrow (i\cancel{\partial} - m_i - q_i\cancel{A})\psi_j = 0$$

$$\partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi_j)} - \frac{\partial \mathcal{L}}{\partial\psi_j} = 0$$

$$⇝ \partial_\mu(\overline{\psi}_j i\gamma^\mu) - \overline{\psi}_j(-m_i - q_i\cancel{A}) = 0$$

$$\Longleftrightarrow \overline{\psi}_j(i\overleftarrow{\cancel{\partial}} + m_i + q_i\cancel{A}) = 0$$

Why for one current a minus sign and for the other not?

b) $$\partial_\mu J_Q^\mu = \partial_\mu \sum_i \underbrace{q_i\overline{\psi}_i\gamma^\mu\psi_i}_{=j_i^\mu} = \sum_i \partial_\mu j_i^\mu$$

We are going to prove $\partial_\mu j_i^\mu = 0$:

$\cancel{A}$ commutes w/ ψ_i?

$$\partial_\mu j_i^\mu = \partial_\mu(q_i\overline{\psi}_i\gamma^\mu\psi_i) = q_i\overline{\psi}_i\overleftarrow{\cancel{\partial}}\psi_i + q_i\overline{\psi}_i\overrightarrow{\cancel{\partial}}\psi_i$$

$$\overset{a)}{=} q_i\overline{\psi}_i\left(-\frac{1}{i}(m_i+q_i\cancel{A})\right)\psi_i + q_i\overline{\psi}_i\left(\frac{1}{i}(m_i+q_i\cancel{A})\right)\psi_i$$

✓

$$=0$$

In lecture different way w/ $\frac{\partial}{\partial(\partial_\mu A_\nu)}$ photon field? ⇝ two ways

Thus $$\partial_\mu J_Q^\mu = \sum_i \partial_\mu j_i^\mu = 0$$

c) As proven, we don't need the sum for the current to be conserved. Each j_i^μ is conserved seperately, meaning that each fermion itself has an conserved current/quantity. With $j_i = (\rho, \vec{j})$, this means that each charge (e.g. e^-, μ^- etc.) is conserved seperately.

Is j_i^μ a particle no. that is conserved or charge?