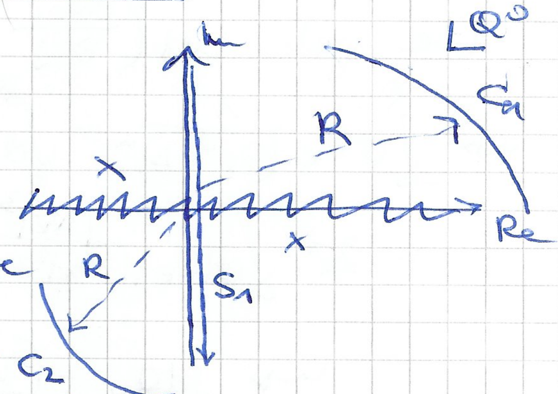
29.11.2017 Theoretical Particle Physics Homework 8 Marius Zäncke

1) a) $I(\Delta,n) = \int\limits_{-\infty}^{\infty} \frac{d^4Q}{(2\pi)^4} \frac{1}{(Q^2-\Delta)^n}$



Introducing an $+i\epsilon$ prescription in the denominator, the poles are located somewhere as sketched.
We thus know from the residue theorem:

$$0 = I(\Delta,n) + \int\limits_{S_1} \frac{d^4Q}{(2\pi)^4} \frac{1}{(Q^2-\Delta)^n} + \int\limits_{C_1} \frac{d^4Q}{(2\pi)^4} \frac{1}{(Q^2-\Delta)^n} + \int \frac{d^4Q}{(2\pi)^4} \frac{1}{(Q^2-\Delta)^n}$$

where S_1 is the path from $+i\infty$ to $-i\infty$ and C_1, C_2 are the arcs sketched in the graph. The contribution from the arcs vanishes, as for $n=1$:

what about the $\vec{Q}^2$ in the denominator? → take into the pole $\Delta_1(\Delta)$

$$I \sim \int\limits_{C_1} dQ^0 \frac{1}{Q^{02}-\Delta'} = \int\limits_0^{\pi/2} d\varphi \frac{1}{r^2e^{2i\varphi}-\Delta'} rie^{i\varphi} \qquad (\Delta' = \Delta + \vec{Q}^2)$$

$Q^0 = re^{i\varphi} \rightarrow Q^{0\prime}(\varphi) = ire^{i\varphi}$

$$\rightarrow |I| \sim \left| \int\limits_0^{\pi/2} d\varphi \frac{ire^{i\varphi}}{r^2e^{2i\varphi}-\Delta'} \right| \leq \int\limits_0^{\pi/2} d\varphi \left| \frac{ire^{i\varphi}}{r^2e^{2i\varphi}-\Delta'} \right|$$

$$= \int\limits_0^{\pi/2} d\varphi \frac{r}{|r^2e^{2i\varphi}-\Delta'|} \leq \int\limits_0^{\pi/2} d\varphi \frac{r}{|r^2-\Delta'|}$$

$$|r^2e^{2i\varphi}-\Delta'| \geq ||r^2e^{2i\varphi}|-|\Delta'|| = |r^2-\Delta'|$$

$$\leq \frac{\pi}{2} \max_{0\leq\varphi\leq\pi/2} \frac{r}{r^2-\Delta'} \xrightarrow{r\to\infty} 0$$

and thus: $I(\Delta,n) = -\int\limits_{S_1} \frac{d^4Q}{(2\pi)^4} \frac{1}{(Q^2-\Delta)^n}$

Parametrizing the imaginary way along the axis and reversing the direction of integration, we find: $Q^0 = iQ_E^0$, $\vec{Q} = \vec{Q}_E$

$$\rightarrow I(\Delta,n) = i \int\limits_{-\infty}^{\infty} \frac{d^4Q_E}{(2\pi)^4} \frac{1}{(-Q^{02}-\vec{Q}^2-\Delta)^n}$$

$$= \frac{(-1)^n i}{(2\pi)^4} \int\limits_{-\infty}^{\infty} d^4Q_E \frac{1}{(Q_E^2+\Delta)^n}, \quad Q_E^2 = Q_E^{0\,2} + \vec{Q}_E^{\,2}$$

b) With $Q_E = |Q_E|\,(\sin\omega \sin\phi \sin\vartheta, \cos\omega \sin\phi \sin\vartheta, \cos\phi \sin\vartheta, \cos\vartheta)$

we get $\det\left(\frac{\partial Q_{E i}}{\partial x_j}\right) = |Q_E|^3 \sin\phi \sin^2\vartheta$

$$\rightsquigarrow \int_{-\infty}^{\infty} d^4Q_E \frac{1}{(Q_E^2+\Delta)^n} = \int_0^{\infty} d|Q_E| \int_0^{2\pi} d\omega \int_0^{\pi} d\phi \int_0^{\pi} d\vartheta\, |Q_E|^3 \sin\phi \sin^2\vartheta \frac{1}{(|Q_E|^2+\Delta)^n}$$

$$= 2\pi^2 \int_0^{\infty} d|Q_E| \frac{|Q_E|^3}{(|Q_E|^2+\Delta)^n}, \text{ as } \int d\Omega_4 = \int_0^{2\pi} d\omega \int_0^{\pi} d\phi \int_0^{\pi} d\vartheta \sin\phi \sin^2\vartheta$$

$$\rightsquigarrow \int d\Omega_4 = (2\pi)\left(\int_0^{\pi} d\phi \sin\phi\right)\left(\int_0^{\pi} d\vartheta \sin^2\vartheta\right)$$

$$= (2\pi)\left[-\cos\phi\right]_0^{\pi}\left[\frac{\vartheta - \sin\vartheta\cos\vartheta}{2}\right]_0^{\pi} = 2\pi^2$$

✓

Why is ω that one going from 0 to 2π?

$\rightsquigarrow (\sin\omega, \cos\omega, \ldots)$

$\Rightarrow$ from 0 to 2

c)

$$I(\Delta,n) = \frac{(-1)^n i}{(2\pi)^4}(2\pi^2)\int_0^{\infty} d|Q_E| \frac{|Q_E|^3}{(|Q_E|^2+\Delta)^n}$$

$$= \frac{(-1)^n i}{8\pi^2}\int_0^{\infty} dz \frac{|Q_E|^3}{2|Q_E|} \frac{1}{(z+\Delta)^n} \quad \text{using } z = |Q_E|^2, \quad \frac{dz}{d|Q_E|} = 2|Q_E|$$

$$= \frac{(-1)^n i}{16\pi^2}\int_0^{\infty} dz \frac{z}{(z+\Delta)^n}$$

$$= \frac{(-1)^n i}{16\pi^2}\left(\int_0^{\infty} dz \frac{1}{(z+\Delta)^{n-1}} - \Delta \int_0^{\infty} dz \frac{1}{(z+\Delta)^n}\right)$$

$$= \frac{(-1)^n i}{16\pi^2}\left(-\frac{1}{n-2}\frac{1}{(z+\Delta)^{n-2}}\Big|_0^{\infty} - \Delta\left(\frac{-1}{n-1}\right)\frac{1}{(z+\Delta)^{n-1}}\Big|_0^{\infty}\right)$$

$$\overset{n>2}{=} \frac{(-1)^n i}{16\pi^2}\left(\frac{1}{n-2}\Delta^{2-n} - \frac{\Delta}{n-1}\Delta^{1-n}\right)$$

$$= \frac{(-1)^n i}{16\pi^2}\left(\frac{(n-1)-(n-2)}{(n-1)(n-2)}\Delta^{2-n}\right) = \frac{(-1)^n i}{16\pi^2}\frac{\Delta^{2-n}}{(n-1)(n-2)}$$

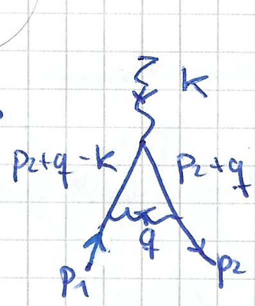
✓

What about $n=2$?

$\rightsquigarrow$ pole, see next sheet

✓ more contributions to the wave fct. renorm. (mass renorm. in fermion case)

2) We considered the process (vertex correction leading order)



✓ Had $\pm xyk^2$ in lecture? +

and had $I^\mu = 2e^3 \bar u(p_2) \int \frac{d^4Q}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \, \frac{2mP^\mu(x+y-(x+y)^2)+\gamma^\mu(\dots)}{D^3} u(p_1)$

where $P^\mu = p_1^\mu + p_2^\mu$, $D = Q^2 - m^2(x+y)^2 - xyk^2$

$k = p_2 - p_1$

there are many gordon decomp.?

First, we derive the necessary gordon decomposition: $(\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu,\gamma^\nu])$

$$\bar u(p_2)\, i\sigma^{\mu\nu} k_\nu u(p_1) = \bar u(p_2)\left(-\tfrac12\right)(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)(p_{2\nu}-p_{1\nu})u(p_1)$$

$$= -\tfrac12 \bar u(p_2)\{\gamma^\mu \not p_2 - \gamma^\mu \not p_1 - \not p_2\gamma^\mu + \not p_1 \gamma^\mu\} u(p_1)$$

$$\overset{\substack{2g^{\mu\nu} = \{\gamma^\mu,\gamma^\nu\} \\ = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu}}{=} -\tfrac12 \bar u(p_2)\{(2g^{\mu\nu} - \gamma^\nu\gamma^\mu)p_{2\nu} - \gamma^\mu\not p_1 - \not p_2\gamma^\mu + p_{1\nu}(2g^{\mu\nu}-\gamma^\mu\gamma^\nu)\}u(p_1)$$

$$= -\tfrac12 \bar u(p_2)\{2p_2^\mu - \not p_2\gamma^\mu - \gamma^\mu\not p_1 - \not p_2\gamma^\mu + 2p_1^\mu - \gamma^\mu\not p_1\}u(p_1)$$

$$= -\tfrac12 \bar u(p_2)\{2(p_2^\mu+p_1^\mu) - m\gamma^\mu - m\gamma^\mu - m\gamma^\mu - m\gamma^\mu\}u(p_1)$$

$$= -\bar u(p_2)\{P^\mu - 2m\gamma^\mu\}u(p_1) \quad \text{using } (\not p - m)u(p) = 0 = \bar u(p)(\not p - m)$$

Consider: $2e^3 \bar u(p_2)\{2mP^\mu(x+y-(x+y)^2)\}u(p_1)$

$$= 2e^3\bar u(p_2)\{2m(x+y-(x+y)^2)[-i\sigma^{\mu\nu}k_\nu + 2m\gamma^\mu]\}u(p_1)$$

As we want to achieve a form like

$$I^\mu = ie\bar u(p_2)\left(\gamma^\mu F_1 + i\frac{\sigma^{\mu\nu}k_\nu}{2m}F_2\right)u(p_1),$$

we will absorb the term above $\sim\gamma^\mu$ in F_1 in addition to $(\dots)$.

Defining $F_2 = 2ie^2(4m^2)\int\frac{d^4Q}{(2\pi)^4}\int_0^1 dx\int_0^{1-x}dy \, \frac{(x+y-(x+y)^2)}{D^3}$, we get

$$I^\mu_{F_2} = -2ie^3\bar u(p_2)(2m)\sigma^{\mu\nu}k_\nu \int d^4Q \int_0^1 dx \int_0^{1-x} dy \, \frac{(x+y-(x+y)^2)}{D^3} u(p_1)$$

Which is exactly the desired term.

b) With $F_2 = 8ie^2m^2 \int \frac{d^4Q}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \frac{(x+y-(x+y)^2)}{(Q^2 - m^2(x+y)^2 - xyk^2)^3}$

we get (for $k^2=0$)

$$F_2(0) = 8ie^2m^2 \int_0^1 dx \int_0^{1-x} dy\, (x+y-(x+y)^2) \int \frac{d^4Q}{(2\pi)^4} \frac{1}{(Q^2-\Delta)^3}, \Delta = m^2(x+y)^2$$

c)
$$= 8ie^2m^2 \int_0^1 dx \int_0^{1-x} dy\, (x+y-(x+y)^2) \left(\frac{-i}{16\pi^2} \frac{\Delta^{-1}}{2} \right)$$

$$= \frac{e^2m^2}{4\pi^2} \int_0^1 dx \int_0^{1-x} dy \frac{(x+y-(x+y)^2)}{m^2(x+y)^2}$$

$$= \frac{e^2}{4\pi^2} \int_0^1 dx \int_0^{1-x} dy \left\{ \frac{(x+y)}{(x+y)^2} - 1 \right\}$$

$$= \frac{e^2}{4\pi^2} \left\{ \int_0^1 dx \{\ln(1) - \ln(x)\} - \int_0^1 dx\, (1-x) \right\}$$

$$= \frac{e^2}{4\pi^2} \left\{ -\{x(\log x - 1)\}\Big|_0^1 - \{1 - \tfrac{1}{2}\} \right\}$$

$$= \frac{e^2}{4\pi^2} \left\{ 1 - \tfrac{1}{2} \right\} \overset{\alpha = \frac{e^2}{4\pi}}{=} \frac{\alpha}{2\pi} = 1{,}16171413 \cdot 10^{-3}$$

$$\rightsquigarrow g_e = 2 + 2F_2(0) = 2{,}00232343$$

$\Delta a_\mu = 2{,}90 \cdot 10^{-9}$

$3{,}5\,\sigma$ $\quad a_\mu^{exp} - a_\mu^{theory}$

✓

Here already set $k^2 = 0$?
$\rightsquigarrow$ yes, calculating $F_2(0)$.
Wolle div. for $F_1(k^2)$ prob. not that easy.

✓

Macroscopic ext magn field ... ?
Static photons ...
$\rightsquigarrow$ photon is on-shell, photon is propagating; magnetic field (nucleus e.g.) mediates photon.
$\vec{B} = \vec{\nabla} \times \vec{A}$ and A_μ in Lagrangian ...

Why mass independent? Different for e and μ?
$\rightsquigarrow$ hadronic and weak int. corrections (bigger for muon than for electron)