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$B \rightarrow \gamma l \bar{\nu}$ (Baebke et al., 2011) 1

27.07.2021 In the following, we want to confirm some calculations from <110.3228> (Baebke et al., 2011); see also the corresponding Mathematica NB.

We consider the decay of a B meson with mass m_B and momentum $p_B = m_B v$ into a photon with momentum q , a neutrino with momentum p_ν and a lepton with momentum p_l . The lepton and neutrino will be assumed to be massless ($l = e, \mu$).

In the B meson rest frame, we have

$$E_\gamma = m_B - \underbrace{\frac{m_B^2 + m_{\gamma l \bar{\nu}}^2}{2m_B}}_{\geq \frac{m_B}{2}} \leq \frac{m_B}{2} \quad \left(\text{or } E_\gamma = \underbrace{\frac{m_B^2 - m_{\gamma l \bar{\nu}}^2}{2m_B}}_{\leq \frac{m_B}{2}} \right),$$

↑
Mathematica

where $m_{\gamma l \bar{\nu}}$ is the invariant mass of the $\gamma l \bar{\nu}$ system.

Introducing the abbreviations

$$x_i = \frac{2E_i}{m_B}, \quad i = \gamma, l, \bar{\nu},$$

we find $0 \leq x_i \leq 1$, where the cases $i = l, \bar{\nu}$ are found analogously to the $i = \gamma$ case. Furthermore, we have $E_B = E_\gamma + E_l + E_{\bar{\nu}} = m_B$, so that

$$x_\gamma + x_l + x_{\bar{\nu}} = \frac{2(E_\gamma + E_l + E_{\bar{\nu}})}{m_B} = 2.$$

The amplitude for the decay $B \rightarrow \gamma l \bar{\nu}$ can be written as

Two upper indices?
in the paper
certainly wrong!

$$A(B \rightarrow \gamma l \bar{\nu}) = \frac{G_F V_{cb}}{\sqrt{2}} \langle l \bar{\nu} \gamma | \bar{l} \gamma_\mu (1 - \gamma_5) V \cdot \bar{u} \gamma^\mu (1 - \gamma_5) b | B \rangle.$$

The photon can be emitted either from the final-state lepton or from one of the constituents of the B meson.

In order to make this explicit, we rewrite the matrix element using the electromagnetic current

$$j_{em}^\mu = \sum_q Q_q \bar{q} \gamma^\mu q + Q_e \bar{l} \gamma^\mu l.$$

LHS upper and RHS lower side
obviously wrong in paper!

To first order in e.m. and to all orders in the strong interactions

we have $(i \not{D} \Gamma - i \not{D} \Gamma - Q_{2q} e A^\mu \Gamma)_{an}$ for QED covariant derivative

$$\langle l \bar{\nu}_\gamma | \bar{u}_{\gamma\mu} (1-\gamma_5) \nu \cdot \bar{u}_{\gamma\mu} (1-\gamma_5) b | B^- \rangle$$

∇?

$$= -ie E_\nu^* \left[\langle l \bar{\nu} | \bar{u}_{\gamma\mu} (1-\gamma_5) \nu | 0 \rangle \right.$$

$$\times \int d^4x e^{iq \cdot x} \langle 0 | T \{ j_{em}^\nu(x) [\bar{u}_{\gamma\mu} (1-\gamma_5) b] (0) \} | B^- \rangle$$

$$+ \int d^4x e^{iq \cdot x} \langle l \bar{\nu} | T \{ j_{em}^\nu(x) [\bar{e}_{\gamma\mu} (1-\gamma_5) \nu] (0) \} | 0 \rangle$$

$$\times \langle 0 | \bar{u}_{\gamma\mu} (1-\gamma_5) b | B^- \rangle \Big], \quad (*)$$

Why the factor of $-i$? In particular, this is opposite to the factor of $+i$ from the Khodjamirian paper?! Also, no relative sign between the two diagrams here

where the first term corresponds to the ^{photon} emission from the meson constituents, whereas the second term describes the emission from the lepton. The latter piece can be calculated exactly

using

$$\langle 0 | \bar{u}_{\gamma\mu} (1-\gamma_5) b | B^-(p) \rangle = -i f_B p_\mu$$

∇?

Why other sign for the matrix element here than in Khodjamirian paper?

and Feynman rules. Moreover, we define the hadronic tensor

$$T^{\mu\nu}(p, q) = -i \int d^4x e^{iq \cdot x} \langle 0 | T \{ j_{em}^\nu(x) [\bar{u}_{\gamma\mu} (1-\gamma_5) b] (0) \} | B^- \rangle,$$

so that the matrix element (*) becomes

$$(*) = -ie E_\nu^* \left[\bar{u}_{\gamma\mu} (1-\gamma_5) \nu \left[T^{\nu\mu}(p, q) \right] \right.$$

$$\left. - i f_B p_\mu \int d^4x e^{iq \cdot x} \langle l \bar{\nu} | T \{ j_{em}^\nu(x) [\bar{e}_{\gamma\mu} (1-\gamma_5) \nu] (0) \} | 0 \rangle \right]$$

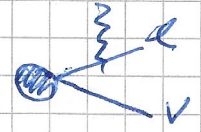
Note that out-gauge contributions are discarded by $\int d^4x e^{iq \cdot x}$

transforms the matrix element in position space to mom. space $\hat{=} Q_e \bar{e} \gamma^\nu l$

$$\hat{=} \langle l \bar{\nu}_\gamma | [\bar{e}_{\gamma\mu} (1-\gamma_5) \nu] (0) | 0 \rangle$$



B → γ l ν (Beneke et al., 2001) 2

Using Feynman rules for the diagram , the remaining leptonic piece is found to be

be $\int d^4x \bar{l}(x) \gamma^\mu l(x) \int d^4y \bar{\nu}(y) \gamma^\nu \nu(y) \int d^4z \bar{q}(z) \gamma^\rho q(z)$

$$= Q_e \bar{u} \gamma^\nu \frac{i(\not{p} + \not{q})}{(p+q)^2} \gamma^\rho (1-\gamma_5) v_\nu (p+q)^\mu$$

$$= i Q_e \bar{u} \gamma^\nu \frac{1}{p+q} (p+q)^\rho (1-\gamma_5) v_\nu$$

$$= i Q_e \bar{u} \gamma^\nu (1-\gamma_5) v_\nu$$

Note (29.09.2022): We forgot to take the integral over d^4x into account; however, it's probably okay to just calculate the expression in mass space, where there is no $\delta^4(x-y)$.

Update (02.12.2021): including the lepton mass, we have $Q_e \bar{u} \gamma^\nu \frac{i(\not{p} + \not{q} + m_l)}{(p+q)^2 - m_l^2} \gamma^\rho (1-\gamma_5) v_\nu \times (p+q)^\mu$

So that $= Q_e \bar{u} \gamma^\nu \left\{ i + \frac{i(\not{p} + \not{q} + m_l) m_l}{(p+q)^2 - m_l^2} \right\} (1-\gamma_5) v_\nu = i Q_e \bar{u} \gamma^\nu (1-\gamma_5) v_\nu + i Q_e \bar{u} \gamma^\nu \frac{(\not{p} + \not{q} + m_l) m_l}{(p+q)^2 - m_l^2} (1-\gamma_5) v_\nu$

$$\langle \mathcal{A} \rangle = e E_\nu^* \left[\bar{u} \gamma_\mu (1-\gamma_5) v_\nu T^{\mu\nu}(p,q) - f_B (i Q_e \bar{u} \gamma^\nu (1-\gamma_5) v_\nu) \right]$$

$= e E_\nu^* \bar{u} \gamma_\mu (1-\gamma_5) v_\nu T^{\mu\nu}(p,q) - i e Q_e f_B \bar{u} \gamma^\nu (1-\gamma_5) v_\nu$

Strong int. knows nothing about the individual lepton momenta, they see the mass transferred away from the flavor changing charged current for the hadronic tensor, we now make the generic ansatz

$T^{\mu\nu}(p,q) = (-i) \left[g^{\mu\nu} a + q^\nu p^\mu b + p^\nu q^\mu c + q^\mu q^\nu d + p^\mu p^\nu e + \epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta f \right]$

and use the Ward identity

$q_\nu T^{\mu\nu}(p,q) = -i f_B p^\mu = -i f_B [g^{\mu\nu} a + q^\nu p^\mu b + p^\nu q^\mu c + q^\mu q^\nu d + p^\mu p^\nu e + \epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta f]$

to find

$q_\nu T^{\mu\nu}(p,q) = (-i) \left[q^\mu a + q^2 p^\mu b + (p \cdot q) q^\mu c + q^2 q^\mu d + (p \cdot q) p^\mu e + \epsilon^{\mu\nu\alpha\beta} q_\nu q_\alpha p_\beta f \right]$

$\Rightarrow -i [a + (p \cdot q) c] = 0, \quad -i (p \cdot q) e = -i f_B$

where we compared the factors for p^μ and q^μ coefficient-wise.

This is in mass space now. Note that propagators $i, e, -1$ are already in front of the whole expression in the paper.

Why only q and p used as independent momenta in $T^{\mu\nu}$? Here $q = p + p_\nu$? Strong int. knows nothing about the individual lepton momenta, they see the mass transferred away from the flavor changing charged current for the hadronic tensor, we now make the generic ansatz

Why include the factor of $(-i)$ here? Just an arbitrary choice? Only effect of the different convention for the covariant derivative is the different sign in the Ward identity? And, implicitly, the matrix element of the decay constant?!

Can also be kept as it was or an alternative calculation

(18) → next page

Thus, we find

$$T^{\nu\tau}(p,q) = (-i) \left[-g^{\nu\tau}(p \cdot q) + q^\nu p^\tau + p^\nu q^\tau + q^\nu q^\tau d + p^\nu p^\tau \frac{f_B}{(p \cdot q)} + e^{\mu\nu\sigma\tau} q_\sigma f \right]$$

$$= (-i) \left[-c(g^{\nu\tau}(p \cdot q) - p^\nu q^\tau) + q^\nu p^\tau b + q^\nu q^\tau d + p^\nu p^\tau \frac{f_B}{(p \cdot q)} + e^{\mu\nu\sigma\tau} p_\sigma q_\tau f \right]$$

Terms $\sim q^\nu$ vanish upon contraction with ϵ^{ν}

$$= (-i) \left[-c m_B^2 (g^{\nu\tau}(v \cdot q) - v^\nu q^\tau) + \frac{v^\nu v^\tau}{(v \cdot q)} f_B m_B + i e^{\mu\nu\sigma\tau} v_\sigma q_\tau \frac{f_B m_B}{i} + (q^\nu\text{-terms}) \right]$$

$$= (-i) \left[(g^{\nu\tau}(v \cdot q) - v^\nu q^\tau) \hat{F}_A(E_\gamma) + \frac{v^\nu v^\tau}{(v \cdot q)} f_B m_B + i e^{\mu\nu\sigma\tau} v_\sigma q_\tau F_V(E_\gamma) + (q^\nu\text{-terms}) \right]$$

So that the remainder consists of two form factors

With the help of the Dirac eq., $(p-q)\gamma_\mu \epsilon_\nu \gamma_\mu (1-\gamma_5) u = 0$

i.e. $m_B \bar{u} \gamma_\mu (1-\gamma_5) u = \bar{u} q_\mu (1-\gamma_5) u$, and

$$m_\nu^2 = (p-q)^2 = m_B^2 - 2(p \cdot q) \Rightarrow p \cdot q = \frac{m_B^2 - m_\nu^2}{2}$$

$$\Rightarrow v \cdot q = \frac{m_B^2 - m_\nu^2}{2m_B} = E_\gamma$$

We can replace

$$(g^{\nu\tau}(v \cdot q) - v^\nu q^\tau) \hat{F}_A(E_\gamma) + \frac{v^\nu v^\tau}{(v \cdot q)} f_B m_B \rightarrow (g^{\nu\tau}(v \cdot q) - v^\nu q^\tau) F_A(E_\gamma) + g^{\mu\nu} f_B$$

where $F_A(E_\gamma) = \hat{F}_A(E_\gamma) + \frac{Q_e f_B}{E_\gamma}$

because

$$\bar{u} \gamma_\mu (1-\gamma_5) u \left[(g^{\nu\tau}(v \cdot q) - v^\nu q^\tau) \frac{Q_e f_B}{E_\gamma} + g^{\mu\nu} f_B \right]$$

$$= \bar{u} \gamma_\mu (1-\gamma_5) u \left[g^{\nu\tau} Q_e f_B - \frac{v^\nu q^\tau}{(v \cdot q)} Q_e f_B + g^{\mu\nu} f_B \right]$$

$$\stackrel{Q_e = -1}{=} \bar{u} \gamma_\mu (1-\gamma_5) u \left[\frac{v^\nu v^\tau}{(v \cdot q)} f_B m_B \right]$$

Alternative $= i \epsilon^{\mu\nu\sigma\tau} m_e Q_e \bar{u} \gamma_\mu (1-\gamma_5) u \gamma_\nu \gamma_\sigma \gamma_\tau (1-\gamma_5) u$

Note that in the BCFG paper they consider $\epsilon(p)$ and $v(p)$, so that one would have

Why do FFs depend on E_γ (tho an q^2 ?)

Why F_V (vector) for the term with ϵ -tensor? Because B is a pseudoscalar so need ϵ -tensor for non-parity violation?

Why F_A (vector) for the term with ϵ -tensor? Because B is a pseudoscalar so need ϵ -tensor for non-parity violation?

Why F_V (vector) for the term with ϵ -tensor? Because B is a pseudoscalar so need ϵ -tensor for non-parity violation?

All of this only works if we insert $Q_e = -1$, also for the photo emission from the lepton "piece". Otherwise, factor Q_e and a sign missing to cancel, and other terms do not vanish.

Have a B^- here? Terms to compare with I. ...

B → γ e ν (Beneke et al, 2001) 3

Then, the photon emission from the lepton is exactly cancelled in (*) ,

$$(*) = e E_\nu^* \bar{u} \gamma_\mu (1-\gamma_5) v \bar{l} \gamma^\mu (1-\gamma_5) l - ie Q_e Q_B \bar{u} \not{\epsilon}^* (1-\gamma_5) v$$

$$= e E_\nu^* \bar{u} \gamma_\mu (1-\gamma_5) v (-i) \left[(g^{\nu\mu} (v \cdot q) - v^\nu q^\mu) F_A(E_\gamma) + g^{\mu\nu} + i \epsilon^{\mu\nu\alpha\beta} v_\alpha q_\beta F_V(E_\gamma) + (g^{\nu\mu} \text{-terms}) \right]$$

$$- ie Q_e Q_B \bar{u} \not{\epsilon}^* (1-\gamma_5) v$$

$\alpha = -1$

$$\downarrow$$

$$= -ie E_\nu^* \bar{u} \gamma_\mu (1-\gamma_5) v \left[(g^{\nu\mu} (v \cdot q) - v^\nu q^\mu) F_A(E_\gamma) + i \epsilon^{\mu\nu\alpha\beta} v_\alpha q_\beta F_V(E_\gamma) + (g^{\nu\mu} \text{-terms}) \right]$$

and the amplitude is entirely determined by the two form factors $F_{A,V}$.

The full amplitude is then given by

$$A(B^- \rightarrow \gamma e \nu) = \frac{G_F V_{ub}}{\sqrt{2}} \left\{ -ie E_\nu^* \bar{u} \gamma_\mu (1-\gamma_5) v \left[(g^{\nu\mu} (v \cdot q) - v^\nu q^\mu) F_A(E_\gamma) + i \epsilon^{\mu\nu\alpha\beta} v_\alpha q_\beta F_V(E_\gamma) + (g^{\nu\mu} \text{-terms}) \right] \right\}$$

In order to calculate the doubly differential decay width, we use the formula

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{8m_B} |M|^2 dE_\gamma dE_e$$

from the PDG.

the (spin-av.) amplitude squared is calculated with Mathematica, where we furthermore use that $q^2=0, p_e^2=0, p_\nu^2=0$

($p = p_e + p_\nu + q$), resulting in

$$|A(B^- \rightarrow \gamma e \nu)|^2 = \frac{8e^2 G_F^2 |V_{ub}|^2}{m_B^2} (p_e \cdot p_\nu) \left[(F_A - F_V)^2 (p_e \cdot q)^2 + (F_A + F_V)^2 (p_\nu \cdot q)^2 \right]$$

(Note that $[\bar{u} \gamma_\mu (1-\gamma_5) v]^\dagger = v^\dagger (1-\gamma_5) \gamma^\mu u = \bar{v} \cdot (1+\gamma_5) \gamma^\mu u$)

What is meant by
However, the
decomposition (2.4)
w/o the replacement
is useful for
calculations, since
it allows us to
assume that the
n-... dies p.v
we transverse
relative to v and q,
respectively, and
that F_V and F_A
can be extracted
from the $\epsilon^{\mu\nu\alpha\beta}$
and $g^{\mu\nu}$ structures
of the hadronic
current? Not really
are purely transverse?
but the parts will
v, F_A indeed
vanish?

update (10.01.2022)

We furthermore have

$$m_{e\nu}^2 = (p_e + p_\nu)^2 = 2(p_e \cdot p_\nu), \quad m_{e\gamma}^2 = (p_e + q)^2 = 2(p_e \cdot q),$$

$$m_{\nu\gamma}^2 = (p_\nu + q)^2 = 2(p_\nu \cdot q), \quad (m_{e\nu}^2 = (p - q)^2 = m_B^2 - 2(p \cdot q)),$$

where in the B meson rest frame, we had/have

$$E_\gamma = \frac{m_B^2 - m_{e\nu}^2}{2m_B}, \quad E_e = \frac{m_B^2 - m_{\nu\gamma}^2}{2m_B}, \quad E_\nu = \frac{m_B^2 - m_{e\gamma}^2}{2m_B},$$

i.e.

$$x_i = \frac{2E_i}{m_B}$$

$$m_{e\nu}^2 = m_B^2 - 2m_B E_\gamma = m_B^2 (1 - x_\gamma),$$

$$m_{\nu\gamma}^2 = m_B^2 - 2m_B E_e = m_B^2 (1 - x_e),$$

$$m_{e\gamma}^2 = m_B^2 - 2m_B E_\nu = m_B^2 (1 - x_\nu).$$

Hence,

$$|A(B \rightarrow e\bar{\nu})|^2 = e^2 G_F^2 m_B^4 |V_{ub}|^2 (1 - x_\gamma) \left[(1 - x_e)^2 (F_A + F_V)^2 + (1 - x_\nu)^2 (F_A - F_V)^2 \right]$$

and ultimately

$$\frac{d\Gamma}{dE_\gamma dE_e} = \frac{\alpha G_F^2 |V_{ub}|^2}{4s_W^2} m_B^3 (1 - x_\gamma) \left[(1 - x_e)^2 (F_A + F_V)^2 + (1 - x_\nu)^2 (F_A - F_V)^2 \right]$$

Note that in the paper, the $(1 - x_e)$ and $(1 - x_\nu)^2$ term get multiplied in exact opposite fashion.

Making use of the assumption that the form factors only depend on the photon energy (and not the lepton energy), we can integrate over E_e , where

$$x_e = \frac{2E_e}{m_B} \implies \frac{dx_e}{dE_e} = \frac{2}{m_B}, \quad E_e = m_B - E_\nu - E_\gamma \stackrel{\leq \frac{m_B}{2}}{\leq} \frac{m_B}{2} - E_\gamma, \quad E_e \leq \frac{m_B}{2}$$

resulting in (with $x_\nu = 2 - x_\gamma - x_e$)

$$\implies x_e \geq 1 - x_\gamma, \quad x_e \leq 1$$

$$\frac{d\Gamma}{dE_\gamma} = \frac{\alpha G_F^2 |V_{ub}|^2}{4s_W^2} m_B^4 (1 - x_\gamma) x_\gamma^3 (F_A^2 + F_V^2).$$