Disclaimer

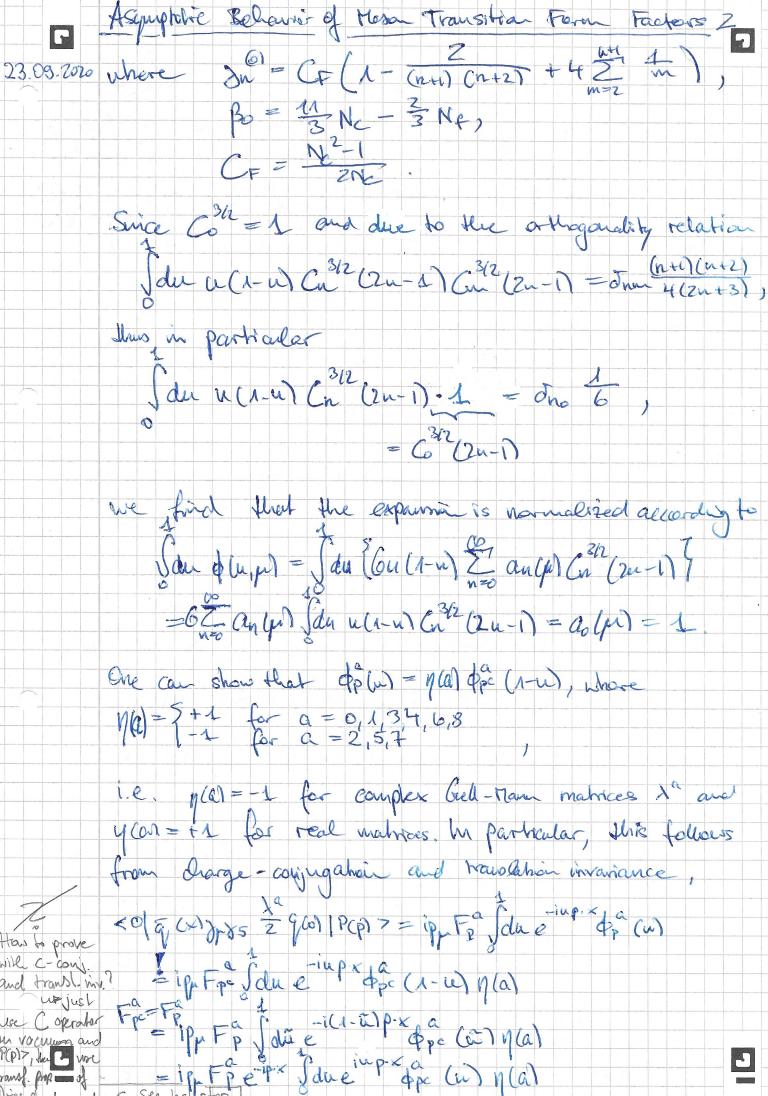
The notes at hand were written during my research period as a PhD student at the University of Bonn. They contain auxiliary calculations to and comments on publications by other authors, which are subject to definite conditions of use; see also the respective article(s) on https://arxiv.org/ linked on the following website. For more information and all my material, check: https://www.physics-and-stuff.com/

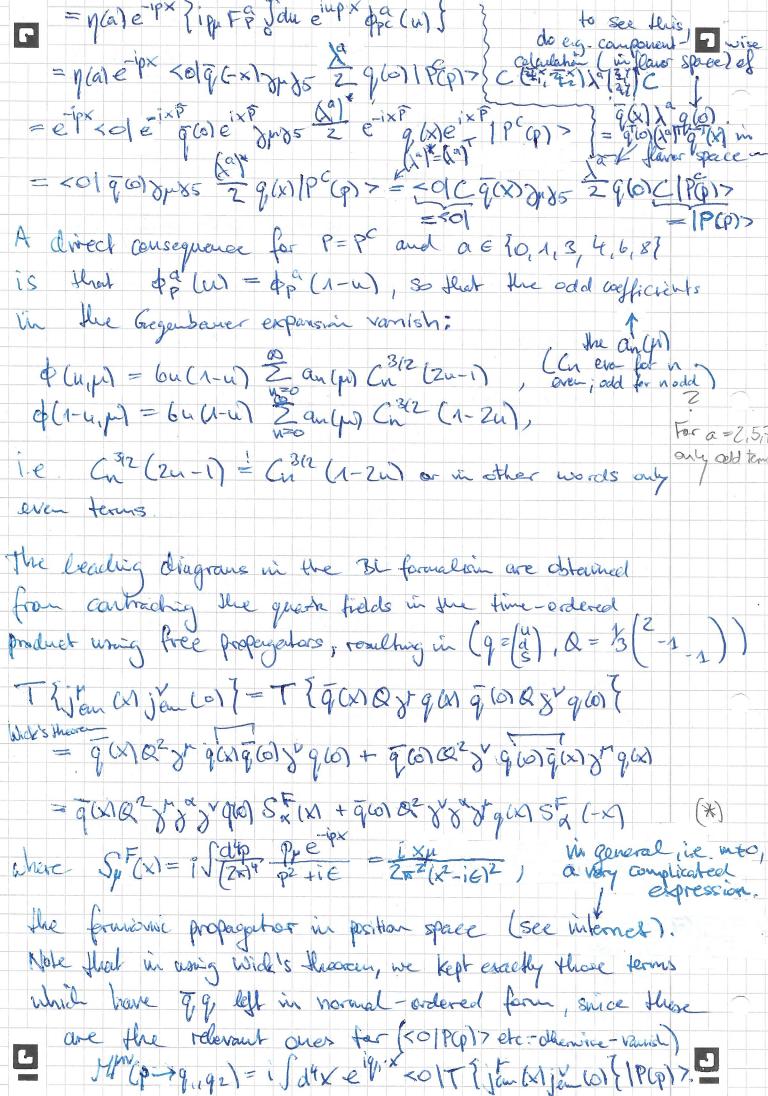
I raise no claim to correctness and completeness of the given material!

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Asymptotic Beliavier of Mesa Transition Form Factor 1 G In the following, we will check some of the Calarlahiers from Marhi's and Peter's paper. (the making (see arXiv: 2004.06127) 21.03.2020 The asymptotic behavior of pseudoscalar transition form factors (describing the decay P-5 (g.) & (g2) has been studied in the literature using an expression along the light Grass X=0. flue light Gare X =0. At leading order, the resulting TFF for the pion Can be expressed as $\frac{2}{2} = \frac{2}{3} \int du \frac{dx}{uq^2 + (u - uq^2 + 0)}$ in terms of the decay condant Fr = 52,28 (19) MeV and the name function of the asymptotic form of the name function reads of (1) = bu (1-1), resulting $F_{ROSESE}(q^2q^2) = -\frac{21\pi}{3q^2} + O(q^{-4})$ for the symmetric limit (= Kincenalic configuration that follows from a strict Operator product expansion) and $F_{\pi 0} g^{2} g^{2} (q^{2} 0) = - \frac{24\pi}{q^{2}} + Q(q^{-4})$ for the right-virtual care Cotton referred to as the Brodsky-Lepayre limit of the singly-virtual TFF). Here, we used frat 4 $\int du [bu(i-u)] = 1$, $\int du [b(u-u)] = 3$. Note that the wave - Inchien approace dready resame higher-order terms, this going beyond a shret OPE. G

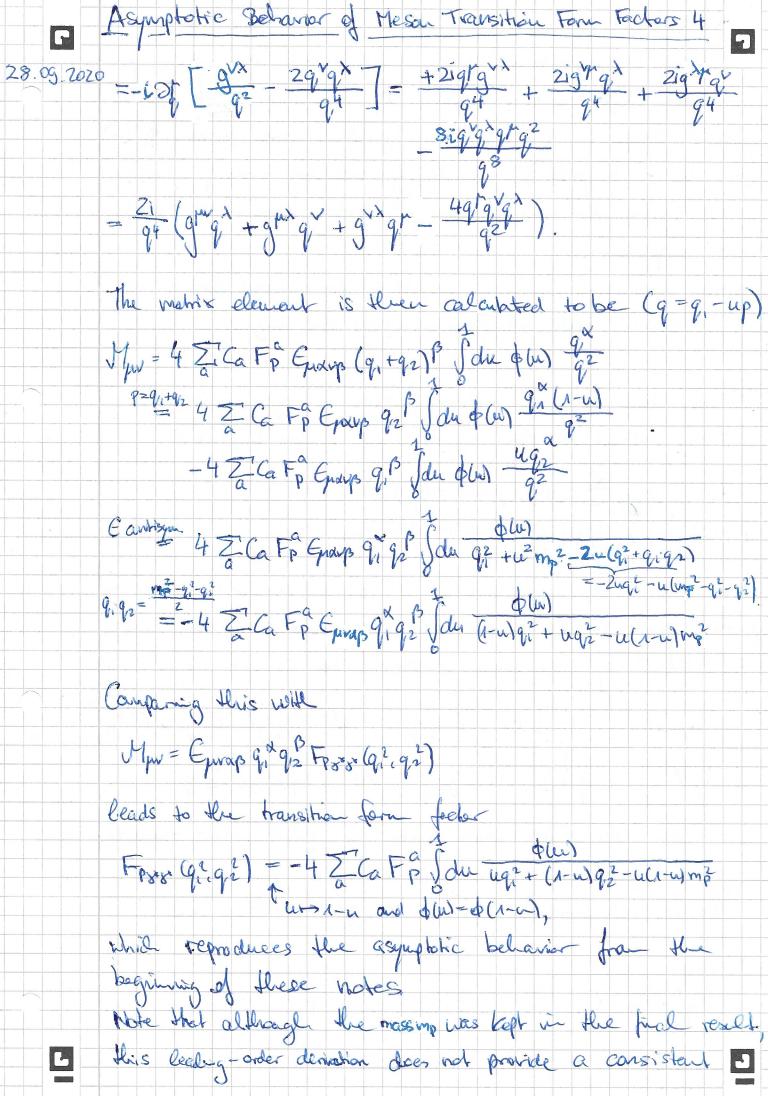
■ The horente Anchie and helicity amplitudes have been worked art (not entirely) in the pathematica - File. 7 We now come to the Bi birt for the transition form factore in more detail. Storting will the pseudoscalar case, where we restrict the analysis to the leading-order result, we define the decay constant Fp via P.S. OF. 10.2000: Need proper pariny an LHS and RHS i.e. 25 for pseudoscalar Why clou't w Var. - a For avial-vector later, also also define Var. - a For avial-vector later, also also define var. - a For avial-vector later, also also define and prod and here haverer, an epsilar tensor appears an RHS will the peace and here the proof questions to calcol will E-tensor (ao for and here how for an avial will E-tensor) (ao for and here the new functions of a scalar particles and here the new functions of a scalar particles and here the new functions of a scalar particles and here the new functions of a scalar particles ME Discussion () will Mahar easts for geen < c \q (x) Jr 35 2 q (c) | P(p) > = ip, Fp Jdu e up x Ja(u), PS that a where a path-ordered gauge factor to council the quarks fields not meroless at a and X has been anithed on the LHS. Using conformal symmetry of RCD, the name functions an be What pathordered gauge Calculated asymptotically, resulting in altor to bunkly 01.10.2020: Note that in the Dre quere helds SU(2) limit (neglecting breaking offds at I and X? $\phi p(u) = bu (1-u) \equiv \phi(u).$ & the LCDAS of all particles Chould are hivially related (equivalent?), for [x,y]=P exp & Sat (x-y), A.H. In fact, we will any consider asymptotic results here ; to the - Wat forl Extent possible, we will write the corresponding have purchia in us needed 200 retain terms of \$(u) from above, Beyond the asymptotic result tocal game syn metry Vitt is The matrix element (x) and this wave protien become gluber backgrand the Scale dependent. However, the conformal analyzir shows that The higher-order terms Can be organized and an expansion Un Greggatoanes polynomials Cu ? -2



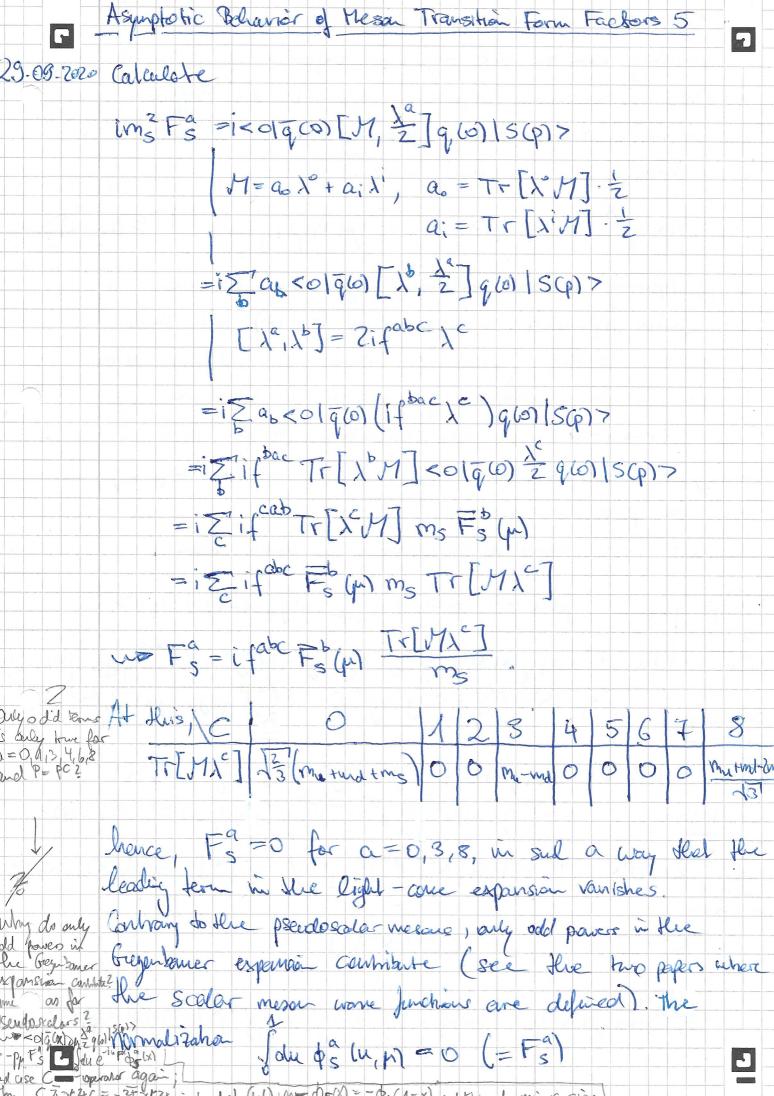


F A Spuptotic Behavior of Meson Tromsition Form Factors 3 27.09-2020 For the remaining Dire Structure, we use $3H_{S}a_{S}v = ghan v + gva_{S}t - gtw a + i e hav B;$ So that $= \overline{\{j_{en} (x) j_{en} (G)\}} = \overline{g}(x)Q^{2}[g_{y} x + g^{v} x] + g^{v} x^{1} - g^{\mu\nu} x + i \mathcal{C}^{\mu\alpha\beta} - \chi + i \mathcal{C}^{\mu\alpha\beta} + i \mathcal{C}^{\mu\alpha\beta} - \chi + i \mathcal{C}^{\mu\alpha\beta} + i \mathcal{C}^{\mu\alpha\beta$ + \overline{q} (a) (22[\overline{g}) \overline{g} + $\overline{g$ $m = \mathcal{M}^{W}(p \rightarrow q_{1}, q_{2}) = i \int d^{4}x e^{i\frac{q_{1}}{p_{1}}} < 0 |\overline{q}(x)Q^{2}[\overline{q}^{M}\partial^{V} + \overline{q}^{M}\partial^{V} - \overline{q}^{M}\partial^{A}]$ + $i\epsilon\mu xy xp = 7q6 S_x(x)$ + $q6) Q^2 Egt x + gx x - g\mu x x - ie^{\mu x p} = 795$ × $q(x) S_x(-x) 1P(p) >$ = i Sdix e 9 × <01 q x 02 [gha y + gva y - gtv x + i Et apps] $\frac{1}{16} = \frac{1}{9} \frac{1}{16} \frac{1}{16}$ $= i \int dx e^{iq_{1}x} < 0 [\overline{q}(x) \otimes^{2} [\overline{q}(x) x + q^{x}x] + q^{x}x] - q^{y}x + i e^{y \alpha y^{2}} \partial_{\beta}y_{5}]$ $\times q(0) S_{\alpha}^{5}(x)$ $+ \overline{q}(x) \otimes^{2} [\overline{q}(x) x + q^{y}x] - q^{y}x + i e^{y \alpha y^{2}} \partial_{\beta}y_{5}]$ $\times q(0) e^{ipx} S_{\alpha}^{5}(-x) [P(p)^{2}]$ $\times q(0) e^{ipx} S_{\alpha}^{5}(-x) [P(p)^{2}]$ 2 h paper: "using ramilational time nd the symmetry She wave fucks All contractions A Deravous pages Jahr clancent richt the same of vectos arrest esult... Vebethert, 14x ral afor need helt maline clarat elfor current venishes? L

= $i \int d^4x e^{q_i x} (2i \epsilon^{\mu \nu}) \times o[\overline{q}(x) \alpha^2 \gamma^2 \delta_5 q(0)] P(p) \rightarrow S_F^{\alpha}(x)$ from translational invariance and the symmetry of the wave function under under $\frac{\mathcal{U}_{ning}}{\mathcal{Q}} = \frac{2}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} = \frac{1}{3} = \frac{1}{3} \times \frac{1}{3},$ $\alpha_i = \operatorname{Tr}\left[\chi \alpha_i^2\right] \cdot \frac{1}{2},$ we find $<01\overline{q}(x)Q^{2}\chi^{\beta}\chi_{5}q(0)|P(p)> = \sum_{\alpha}G_{\alpha}cq\overline{q}(x)\chi^{\beta}\chi_{5}\lambda_{\alpha}q(0)|P(p)>$ $G_{a} = \frac{1}{2} T_{r} [\chi^{a} Q^{2}] = \frac{1}{2} T_{r} [Q^{2} \chi^{a}]$ and hus $M_{\mu\nu} = i \int d^{\mu} x e^{iq_{1}x} \left(2i \mathcal{E}_{\mu} a_{\nu} \beta \right) < 0 | \overline{q} (x) \partial_{x}^{2} x^{\beta} x^{5} q^{(0)} | P(p) > S \not\in (x)$ $= -4i \sum_{q} C_{q} \mathcal{F}_{p}^{a} \left(\overline{q}_{1} + q_{2} \beta \right) \int du \phi(u) \int d^{\mu} x e^{iq_{1} \cdot x} e^{-iu p \cdot x} x \xrightarrow{x} (x)$ At this, $C_3 = \frac{1}{6}$, $C_8 = \frac{1}{6 \cdot 137}$, $C_0 = \frac{2}{3 \cdot 6}$, and all other variable. Vannesh. Since SF (X) = i J (Zx)" Pre-ipx Since Sp (X) = i J (Zx)" P2+iE, we find for the Feynma Prepayator that $\int d^4x \, S_F^h(x) \, e^{iq \cdot x} = i - \frac{q^{\mu}}{q^2}, just by Fourier transforming.$ Furthermore, $\int d^{4}x \, xt \, S_{F} \cos e^{iq \cdot x} = -i \mathcal{D}_{q}^{*} \int d^{4}x \, S_{F}^{*} \cos e^{iq \cdot x}$ $= -i \mathcal{D}_{q}^{*} \int d^{4}x \, S_{F}^{*} \cos e^{iq \cdot x}$ $= -i \mathcal{D}_{q}^{*} \int i \frac{q^{*}}{q^{2}} \int -\frac{q^{*}}{q^{2}} -\frac{2q^{*}}{q^{4}}$ as well as $\int d^{4}x \times x^{4}x^{7} S_{F}(x) e^{iq \cdot x} = -i \partial_{q}^{n} \int d^{4}x \times S_{F}(x) e^{iq \cdot x}$ 9

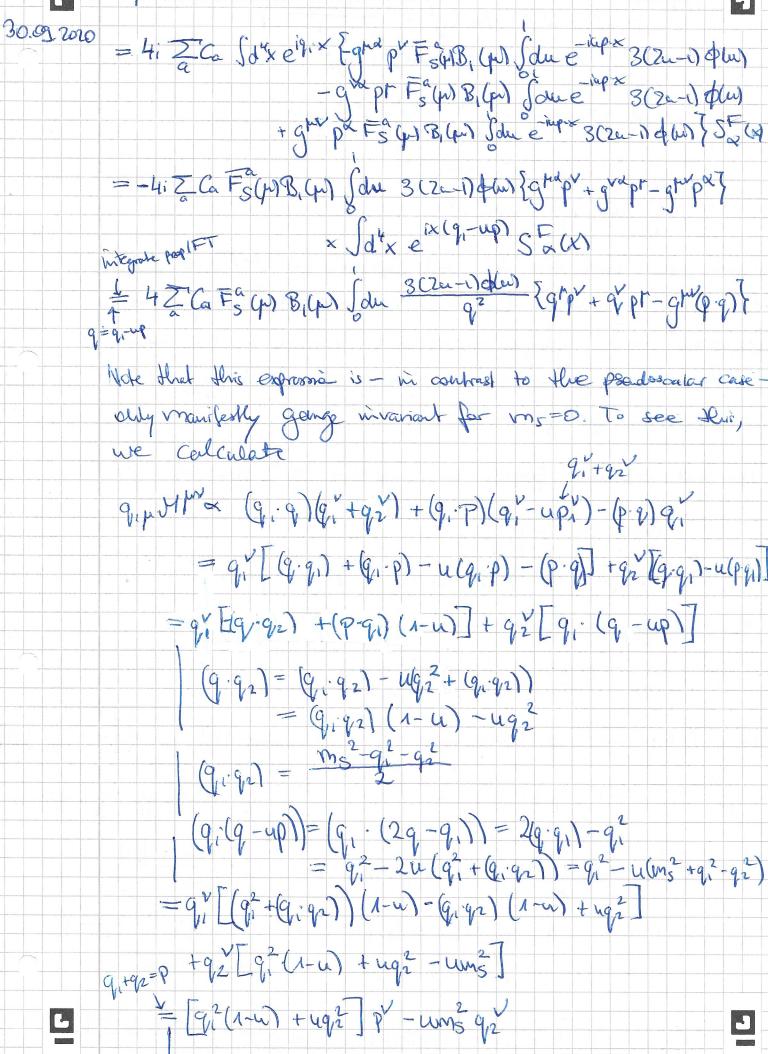


treatment of mass gleets. To this and are would r have to differentiate between the meson momentum p 7 Would appear in the integral of the wave pucking of (u) (in the entry fector); a ceardingly, including terms of U(mi) would require the consideration of Subleading terms in the light core expansion. Moreover, the obtained repult can only be strictly justified from an OPE in the limit in which both phonon virtualities are large. We now turn to scaler mesons, where we equivalently define a decay constant for the vector and scalar current. $<0|\bar{q}(0)\rangle_{\mu} \stackrel{\chi^{\mu}}{=} q(0)|S(p)\rangle = -p_{\mu}F_{S}^{\alpha}$ $<0|\bar{q}(0)|\stackrel{\chi}{=} q(0)|S(p)\rangle = M_{S}F_{S}^{\alpha}(\mu),$ Where the scale dependence in FS (pr) is canceled by the one of the quark masses. In particular, the two decay constants are related by conservation of the vector current, i.e. $\partial_{\mu} V^{\mu \alpha} = i\overline{q} [M, \frac{1}{2}] q, V^{\mu \alpha} = \overline{q} T \frac{1}{2} q, M = \begin{bmatrix} m_{\mu} & o \\ o & m_{d} \\ o & m_{s} \end{bmatrix}$ $(3, A^{r_{1}c_{1}} = iq [\frac{\lambda^{q}}{2}, M] \chi_{5}q, A^{r_{1}c_{2}} = \bar{q}\chi_{5}\chi_{5} + \frac{\lambda^{r_{1}}}{2}q)$ Similarly relates the areal-vector and psoudoscalar - isovector -Current; for non-vanishing quere and lor pion masses, this becomes relevant for the prin decay constant as well.), So that $\Im^{h} \times O[\overline{Q}[0]] \partial_{\mu} \stackrel{\lambda^{h}}{=} Q[O][S(p)] = ip^{2} F_{s}^{a} = im_{s}^{2} F_{s}^{a}$ E Using the divergence from above, we can alknowing

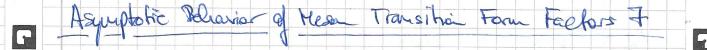


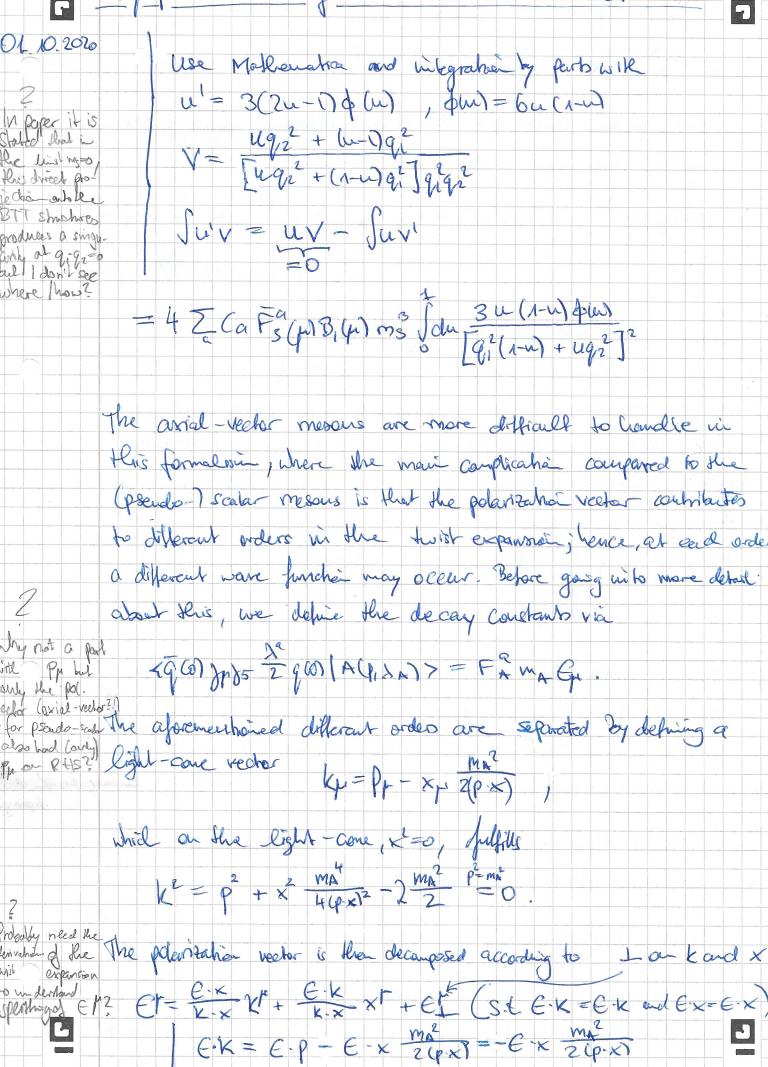
reflects the fact that Fs = 0. Therefore, the first nou-vanishing term in the Gegenbouer expansion of \$s (up) 02 arly FS=0 for 1a=0,3,8 regarding H Undres an unknown Olegebaner coefficient; this coefficient Dormalization OR at Most for Can be made dimensionless by factoring out the scalar decay constant FS. We then write a=0,1,3,4,6,8d 3(2-(2u-1)(=a) to evan terms = CA wanish 3 (2u-1) olu), Mote Lig that bula-in Balp) referring to the adjuntance adjuicent and assuming that a= 0,3,8 are the all illance dage to is a all the Ea (w) while illest that the of Ca all flavor dependence is captured by FS (4). Note that in contrast to the pseudoscelar case, the additional factor the wave for of Qu-1) in the instegral gues rise to an extra mines sign upper Utral-u (see the derivation for the Pseudoscalar so that For Case; this is also consistent with $\phi_s^a(\omega) = -\phi_s^a(1-\omega)$ from before, cald this M some way d written with pencil as a side note). to a - dependent But this is the habit dement We are now ready to coloulate impossible? mesee above $M^{\mu\nu}(p \rightarrow q_{\mu}q_{2}) = i \int d^{\mu} x e^{iq_{1}x} \times O[T[x]e_{\mu}(x)]e_{\mu}(x) [Sq) >$ only a=0,3,8 are relarant; which all have faloury Sendosality Jour ein x x 1 g w 02 [grad v + g va t - gruga + i Enarp Jobs J good S. Fur + g 60 02 [grad v + g va t - gruga - i Erarp Jobs J good S. 5 [-1180]? astal Current vanishes and Jd4x ei9:x x01qcn Q2[gtx v +graft-gtw x2]qco1SFalx1 -eipx q(-x)Q2[gtx y +graft-gtw y2]qco1SFalx1 SQ17 Sleps nom PS = Similar to PS hs additional Minus * 0 A sign 21 Joux eigix x 01 q con 02 [gra 8 + gra 8 - gm 3 2] q con Stack 1 Scp > ~ wore uchie ofs(in) you use the $Q^2 = C_q \lambda^a$, $C_a = \frac{1}{2} Tr[Q^2 \lambda^a]$ =4: Z'Ca Jdtx eigix <01qcx1Egray + grage - grage J 2 (0) (SCP)>SECX)





 $q^{2} = q_{1}^{2} + u^{2}p^{2} - 2u(q_{1}; p)$ = $q_{1}^{2} + u^{2}p^{2} - 2u(q_{1}^{2} + \frac{ms^{2}-q_{1}^{2}-q_{2}^{2}}{2})$ G 7 $= q_{1}^{2} (1 - u) + u^{2} p^{2} - u m_{5}^{2} + u q_{2}^{2}$ $u_{10} = q_1^2 (1 - u) + uq_2^2 = q_1^2 - u^2 p_1^2 + u_{10} + u_{10}^2 = q_1^2 - u^2 p_1^2 + u_{10} + u_{10}^2 + u_{10} + u_{10} + u_{10}^2 + u_{10} + u_{$ $= (q^2 - u^2 p^2 + um_s^2) p^2 - um_s^2 q_2^2$ $= \left(q^2 - u^2 p^2\right) p^2 + um^2 q^2$ Inserting this back with the fiel expression for I Ctr, i.e. with the integral, reveals that we need P=ms=0 for the expression to be gauge invariant (= vanish); see also Mathematica. Analogousty, we could show that governm only vanishes for p²=ms²=0. The goal is now to project this onto the BTT shicking $T_{1}^{\mu\nu} = (q_{1}^{\mu}q_{2}^{\mu})g^{\mu\nu} - q_{2}^{\mu}q_{1}^{\nu}$ $T_{2}^{\mu\nu} = q_{1}^{2}q_{2}^{2}g^{\mu\nu} + (q_{1}^{\mu}q_{2})q_{1}^{\mu}q_{2}^{\nu} - q_{1}^{2}q_{2}^{\mu}q_{1}^{\nu} - q_{2}^{2}q_{1}^{\mu}q_{1}^{\nu}$ and read off the form factors in $\mathcal{M}^{\mu\nu} = \frac{1}{m_s} T_1^{\mu\nu} T_1^{s} + \frac{1}{m_s} T_2^{\mu\nu} T_2^{s}$ (2u-1)(T,) pr + [9] + (u-1) [12) Using Mathematica, we find that Mm = 4 Z Ca F'S Gul B, (p) Jdu 3 C2u - 1) dur (gr pr + grp - gm (p. g)) $\mathcal{F}_{i} = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right)^{2} \right) = \frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} -$ $\begin{cases} 2 = q_1^2 + u^2 p^2 - 2u(q_1 p) = q_1^2 + u^2 m_2^2 - 2u(q_1^2 + \frac{m_2^2 - q_1^2}{2}) \text{ prelactor} \\ q = q_1^2 + u^2 p^2 - 2u(q_1 p) = q_1^2 + u^2 m_2^2 - 2u(q_1^2 + \frac{m_2^2 - q_1^2}{2}) \text{ see also} \\ q = q_1^2 + u^2 p^2 - 2u(q_1 p) = q_1^2 + u^2 m_2^2 - 2u(q_1^2 + \frac{m_2^2 - q_1^2}{2}) \text{ see also}$ $= q_{1}^{2} (1-u) + uq_{2}^{2} + u^{2}m_{3}^{2} - um_{3}^{2} (and m_{3}^{2} = 0) 08.09.000$ $14:24 \circ uve.$ $J_{2}^{-3}(q_{2}^{2},q_{2}^{2}) = 4 \sum_{a} C_{a} F_{s}^{a}(\mu) B_{i}(\mu) m_{s}^{3} \int d\mu \frac{3(2\mu-1)\phi(\mu)[\mu q_{2}^{2}+\mu q_{1}^{2}]}{[q_{1}^{2}(\lambda-\mu)+\mu q_{2}^{2}] q_{2}^{2} q_{2}^{2}}$ L





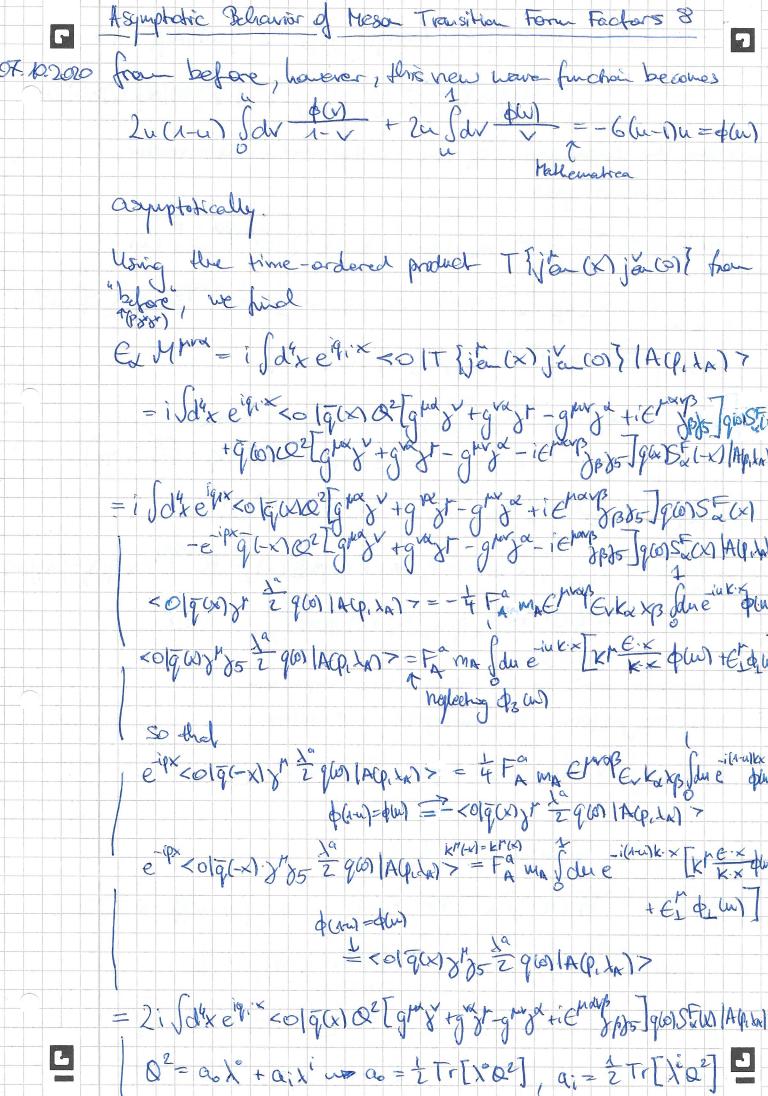
 $\mathbf{r} \quad | \mathbf{k} \cdot \mathbf{x} = \mathbf{p} \cdot \mathbf{x} - \mathbf{x}^2 \quad \frac{\mathbf{m} \mathbf{x}^2}{2(\mathbf{p} \cdot \mathbf{x})} = \mathbf{p} \cdot \mathbf{x} \quad \text{for } \mathbf{x}^2 = \mathbf{o}$ 7 $= \frac{E \times (kr - \frac{ma^2}{2(k - \chi)} \times r) + e^{\mu}_{L}$ this decorposition gives rise to three different wave furching

Ocurring the the asial wector matrix element:

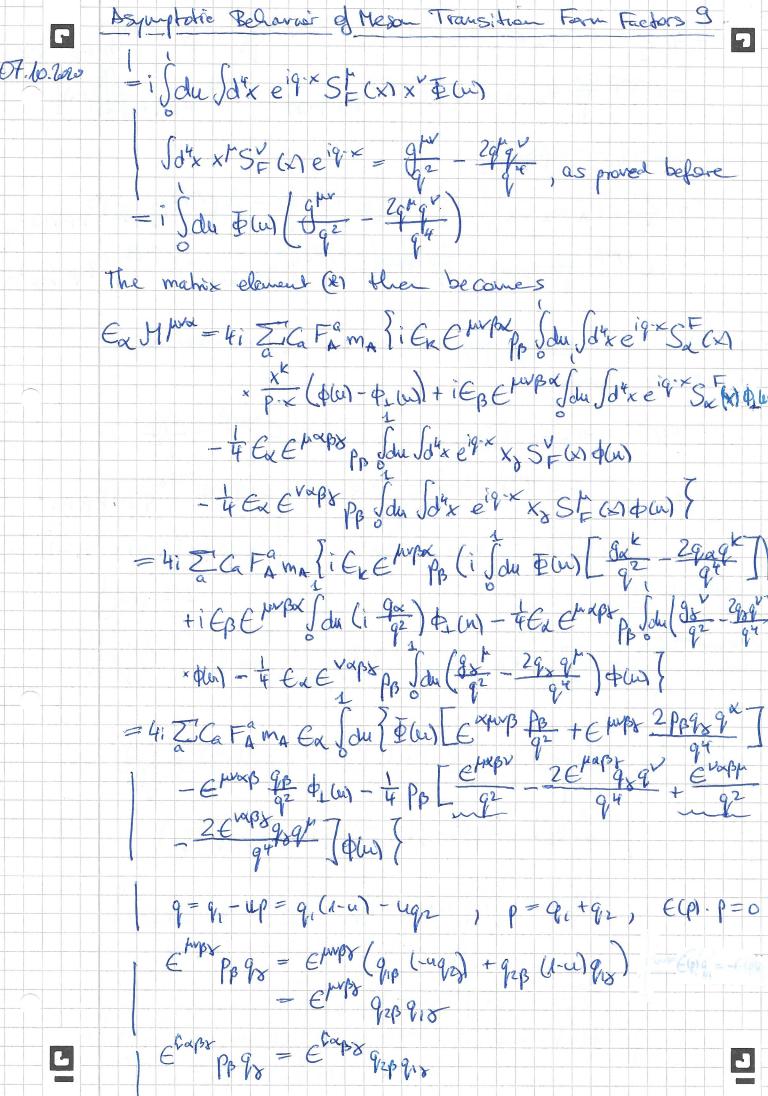
 $x o \overline{q} (x) \gamma \overline{s} = \frac{\lambda^{\alpha}}{2} q (o) |A(q, \lambda)\rangle = F_A^{\alpha} m_A \sqrt{du} e^{-iuk \cdot x} [kr \frac{E \cdot x}{K \cdot x} du)$

- $+ E_{\perp}^{r} \phi_{\perp}(w) \chi \frac{m_{\lambda}^{2} \cdot E \cdot \chi}{2(k \cdot \chi)^{2}} \phi_{3}(w)$
- where \$, will and \$3 Will are of higher twist. In order to obtain a gauge-invariant result for the TFF, Here wave functions should be replaced by so-called handburg -Wilczek relations in terms of the leading twist - 2 distribution amplitudes, which effectively neglects three - poton contributions. In this approximation, one has 1 Mathematica $\Phi_{L}(u) = \frac{1}{2} \int dv \frac{d(v)}{1-v} + \int dv \frac{d(v)}{\sqrt{1-v}} = \frac{3}{2} + 3(u-1)u$
 - = 2 (3 4(1))
- for the asymptotic till from the pseudoscolor mesone. The wave huncha \$2(11) closs not actually contribute due to she antisymmetry Haw is pace related to the of the E-taser-but it could be obtained with similar E-tensor? WT See ales step methods (see reference in paper). Conhaction of e with Stand x
- prepactor of \$2(11) Vanishes. Il contrast to the pseudoscalar are, there is now also a non-vanishing cartribution from the vector matrix element Why not alm any 25?
- < olacity 2 q co) IACP, XA)> = 4 FA MA ENVAPEr Ka XB John E \$w,

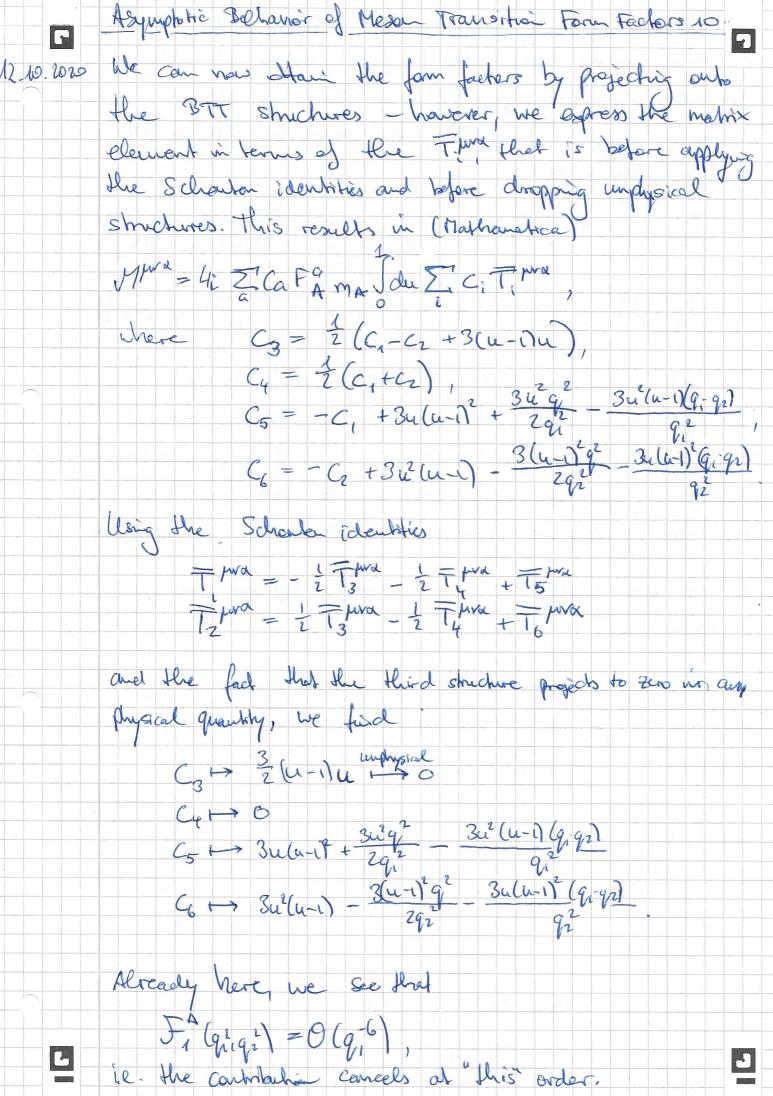
Which is again a twist - 3 Contribution and technically requires Some Fa in A and Vancart and Vancart



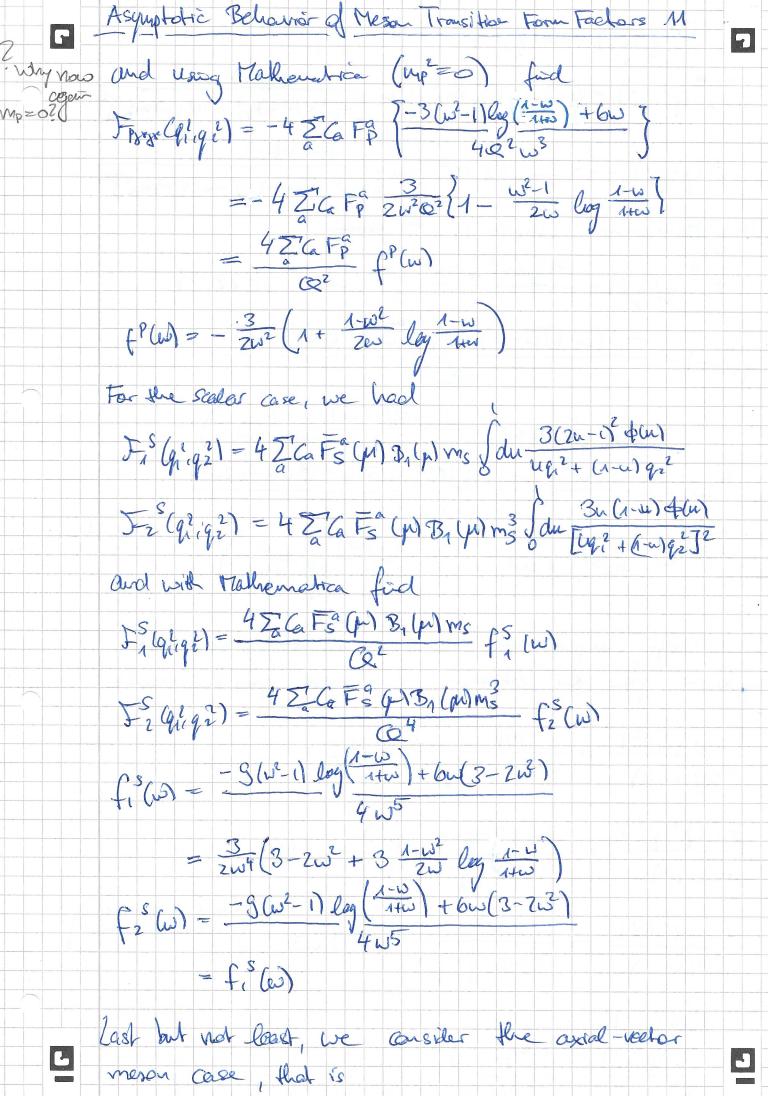
 $=4:\sum_{n}C_{n}\int d^{4}x e^{iq_{n}x} \left\{i\in \mu^{a}\mu^{a}\times O(q(x))\right\} = \frac{\lambda^{n}}{2} \left\{i(x)\left(\lambda_{n}\right)^{2}\right\} = \frac{1}{2}$ + < algest [gtox v + gra r - gra] = 2801 A(q, 1)> SECA Inserting the docan position of the vector and arrial - rector matrix demants gives ELHINA = 4; Z Ca FAMA John Jak eig X [iEmvBod SF CM p=q-up becanse x== on LC - but no $k_{\beta} = p_{\beta} - x_{\beta} \frac{m_{x}^{2}}{2(p \cdot x)}, \quad SF_{\alpha}(x) \propto x_{\alpha}$ $k \cdot x = p \cdot x, \quad E_{\pm \beta} = E_{\beta} - \frac{E \cdot x}{k \cdot x} \left(k_{\beta} - \frac{m_{\alpha}^{2}}{2(k \cdot x)} x_{\beta}\right)$ q=q-up == 4i Zica Fa ma John John John Ste CM Use dution. of epsilon - Jeenson ×[PB p × (qui) - \$, (u)) + EB\$ (u) - 4E VaBY Exp3x, Stappin) Several times $-\frac{1}{4} \in paps \in APB \times SF(x) \neq (u) \{, (*)$ agai hung reglected in the light -core expansion. In Order to perform the integral we define But the open QULAS, eg, FUNXTO5 2 Nome fixed twist $\overline{\Phi}(w) = \int dv \left[\frac{d(v)}{-\phi_1} - \phi_2 \left[\frac{v}{-1} \right] - \frac{2u-1}{4} \phi(w) \right]$ Trathenaticanamely 2, so how tan Here be terms of e. twist 3 on RHS Using integration by parts, we find $\int du \int d^{4}x e^{iq \cdot x} SF(x) \xrightarrow{X} [dw] - d_{\perp}w], q = q, -u \cdot p$ Jan eig Etwa-pin] = [Jar [pin-pin] eig x/o - John Elin [-ix·p] ~ = i Jdu Eur (p.x) C 5



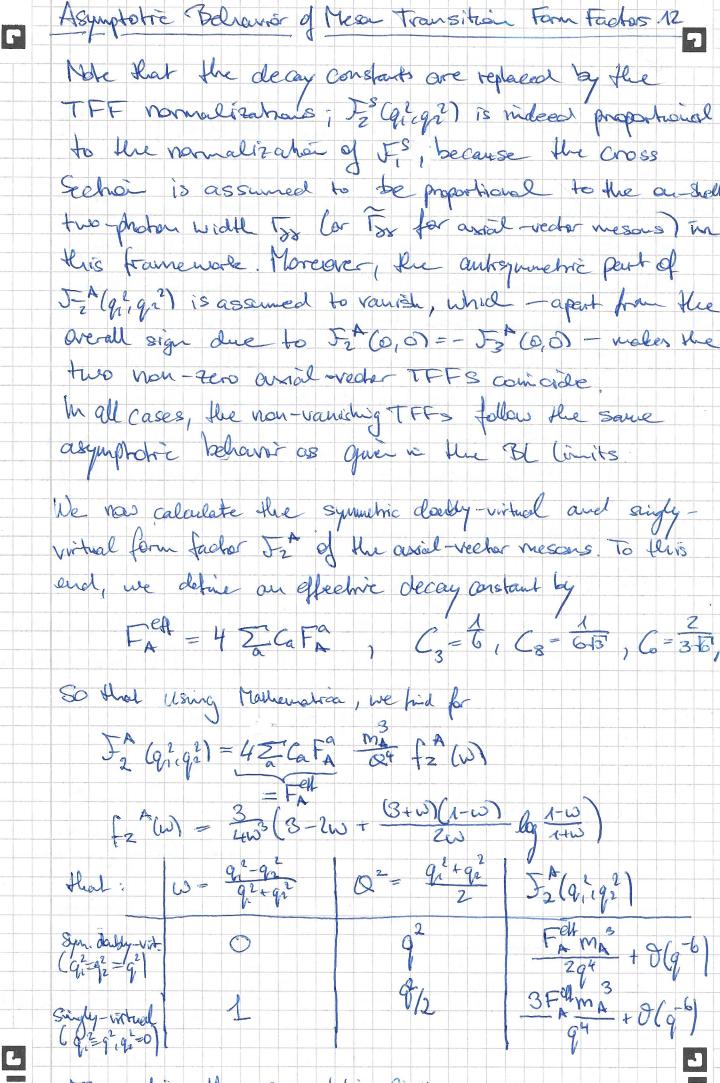
 $\mathbf{r} = 4i \sum_{z} \mathcal{L} \mathbf{r}_{A} \mathbf{r}_{A} \mathbf{r}_{z} \mathcal{L} du \left\{ \mathbf{\Xi} \mathbf{u} \right\} \left\{ \mathbf{\Sigma} \mathbf{r}_{A} \mathbf{r}_{B} \mathbf{r}_{A} \mathbf{$ $E(p)q_1 = -E(p)q_2 1$ = 4i Zi Ca Fa ma Ex Jdu { $\overline{\Phi}(u)$ [$\underline{\epsilon}$ xmp $\underline{f}_{2}^{c} - \underline{\dot{q}}_{1} \underline{\epsilon}$ mps ($q_{1} - q_{2}$) $\dot{q}_{1} \underline{\beta} \underline{g}_{2}$] - $\underline{\epsilon}$ xmp \underline{q}_{2} $\underline{\phi}_{1}$ (u) + $\underline{\dot{q}}_{4}$ $\underline{d}u$) [$\underline{\epsilon}$ xmp q \underline{q}_{1} \underline{g}_{2} + $\underline{\epsilon}$ xmp q \underline{g}_{1} \underline{g}_{2} \underline{g}_{1} (u) + $\underline{\dot{q}}_{4}$ $\underline{d}u$) [$\underline{\epsilon}$ xmp q \underline{g}_{1} \underline{g}_{2} \underline{g}_{2} \underline{g}_{1} (u) + $\underline{\dot{q}}_{4}$ $\underline{d}u$) [$\underline{\epsilon}$ xmp q \underline{g}_{1} \underline{g}_{2} \underline{g}_{2} \underline{g}_{1} (u) + $\underline{\dot{q}}_{4}$ $\underline{d}u$) [$\underline{\epsilon}$ xmp q \underline{g}_{1} \underline{g}_{2} \underline{g}_{2} \underline{g}_{2} \underline{g}_{1} (u) + $\underline{\dot{q}}_{4}$ $\underline{d}u$) [$\underline{\epsilon}$ xmp q \underline{g}_{2} \underline{g}_{1} \underline{g}_{2} \underline{g}_{2} This expression is already gauge invariant, eva for non-zero ma, cus readily diedwel Pip Ex Mura = 4i Zi Ca Fama Ex John [Few Capup 9/1 / 9/2 gi $\frac{e^{\alpha \nu p k}}{e^{-\epsilon \nu p \nu p}} = \frac{e^{\alpha \mu \nu p}}{e^{\alpha \nu p}} \frac{g_{\mu \nu} g_{\mu p} (c^{\mu})}{g_{\mu}} \phi_{\mu \nu} + \frac{i}{2g_{\mu}} \phi_{\mu \nu} e^{\alpha \nu p s} (g_{\mu}) g_{\mu p} g_{\mu s} - \frac{i}{2} (g_{\mu}) g_{\mu s} - \frac{i}{2} (g_{\mu s}) g_{\mu s} - \frac{i}{2} (g_{\mu s}$ Mathenalica >= 0 Allematively = 4: 27 C Fa my Ex E Mars que for 2 (3u²(u-u)) How to trallematical = 4: 27 C FA my Ex E Mars que grande du de (292) get have? (=0 to Suchian of $\frac{2}{2u}\left(\frac{3u^{2}(u-1)}{2q^{2}}\right) = \frac{6u(u-1) + 3u^{2}}{2q^{2}} = \frac{3u^{2}(u-1)}{2} \frac{2u(q^{2})}{2}$ integral will Mathematrica ?! Deriving the give Subà $\frac{2}{2}(q^2) = 2q(\frac{2}{2}(q)) = -2q(q) + q_2)$ hores but how to get there? $= \frac{3u^{2}}{2q^{2}} + \frac{q}{q^{4}} \left[3u(u-i)q^{2} - \frac{3u^{2}}{2u}(u-i)(-2(q,p)) \right]$ $= \frac{3u^{2}}{2q^{2}} + \frac{q}{q^{4}} \left[3u(u-i)q^{2} - \frac{3u^{2}}{2u}(u-i)(-2(q,p)) \right]$ $= \frac{3u^{2}}{2q^{2}} + \frac{1}{q^{4}} \left\{ 3u(u-1)(q,q_{1}) \right\}$ $= \frac{3u^{2}}{2q^{2}} + \frac{1}{q^{4}} \left\{ 3u(u-1)(q,q_{1}) \right\}$ $= \frac{4u}{4} + \frac{3}{2}u$ $= \frac{3}{2}u(n-u) + \frac{3}{2}u = \frac{3}{2}u^{2}$ C



For Jus (qi', qi'), one has to perform further steps, similar to the scalar case (in paper: expressing all scalar 27 Haw to ans products in terms of Qiq2), q², and Saige as well as into integration by parts.). Recalling the prefactor i/ma² in Max form. one ultricately fids $\mathcal{F}_{1}^{1}\left(q_{1}^{2}\right) = 4 \sum_{a} C_{a} \mathcal{F}_{A}^{a} m_{A}^{3} \left[du \left[uq_{1}^{2} + (n-u)q_{2}^{2} - u(n-u)m_{A}^{2} \right]^{2} \right]$ $\overline{F_3(q_1^2)} = -4 \overline{Z_a} \overline{F_a} \frac{1}{m_a} \int_{du} \frac{(n-u) \phi(u)}{[uq_1^2 + (1-u)q_2^2 - u(n-u)m_a^2]^2}.$ The singly-virtual case F2 (0,92) lamong others) does not Converge; this begandlunic end-paint singularity has been observed before. Since the fourd expression Maria is gauge invariant and free of kinematric surgerlarities even for finite my, it is meaningful to keep the assial weeter mess in the first results The analogous" analysis of tenser mesons is left out have due to many more complications. We would to summarize the results in terms of their scales in the average proton virtualities Q2 and the asymmetry parameter W; $Q^2 = \frac{q_1^2 + q_2^2}{2}$, $W = \frac{q_1^2 - q_2^2}{q_1^2 + q_2^2}$ $u = u (q_1^2 + q_2^2) = q_1^2 - q_2^2 \iff q_2^2 = \frac{1 - \omega}{1 + \omega} q_1^2$ $\sum (Q^2 = \frac{1}{2} Q_1^2 \left(1 + \frac{1 - \omega}{1 + \omega} \right) = \frac{Q_1^2}{1 + \omega} \iff Q_1^2 = (1 + \omega) Q^2$ ~~ Q2 = (1-w) Q2 We start will the pseudoscalar case, $\mathbf{L} = \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} - 4 \frac{1}{2} \frac$ Ģ

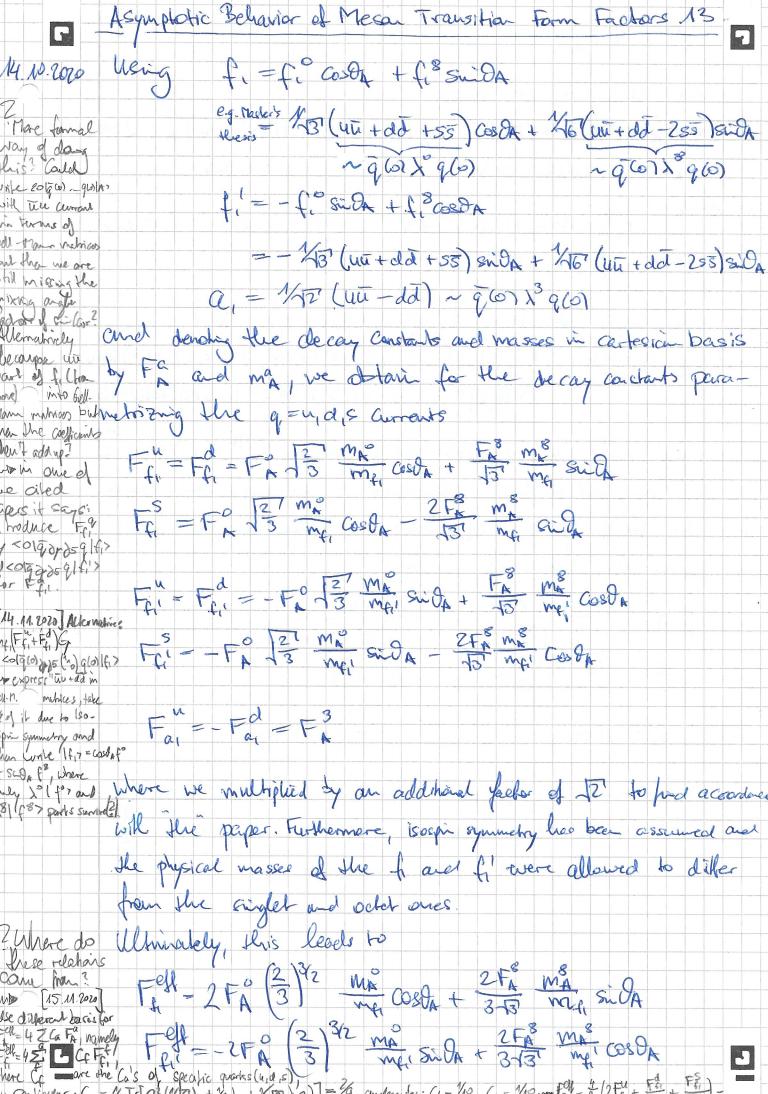


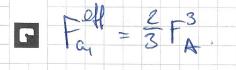
 $\Box = \int_{A}^{A} (q_{1}^{2} q_{2}^{2}) = \partial(q_{1}^{-6}) \longrightarrow \int_{A}^{A} (q_{1}^{2} q_{2}^{2}) = \partial(\phi^{-6})$ 7 Frank (girgi) = 4 ZiCa FA ma Solu Figit (A-wigit - uluwing Je $F_3 = -4 \sum_{\alpha} C_{\alpha} F_{\alpha}^{\alpha} m_{\alpha}^{\beta} \int du \frac{(1-1)\phi(u)}{[uq_1^2 + (1-u)q_2^2 - u(1-u)m_{\alpha}^2]^2}$ Using Mathematica (My =0), we obtain Fi (quique = 42 Ca Fama fi (N) rie 22,33 $f_{2}^{A}(\omega) = \frac{-3(\omega^{2} + 2\omega - 3)\log(\frac{1-\omega}{1+\omega}) + 6\omega(3-2\omega)}{8\omega^{4}}$ $=\frac{3}{4\omega^{3}}\left(3-2\omega+\frac{(3+\omega)(1-\omega)}{2\omega}\log\frac{1-\omega}{1+\omega}\right)$ $\int_{-3}^{-3}\left(\omega^{2}-2\omega-3\right)\log\left(\frac{1-\omega}{1+\omega}\right)-O(1+\omega)$ 8 wt Note the add. Note the add. $(3 - w)(1+w) = \frac{3}{4w^3}(3+2w) + \frac{(3-w)(1+w)}{2w} \log(1+w)$ We briefly Compare the El scalinge to "the quark model approach for the literature. Said madel gives the results $\frac{F_{psys}(q_{1}^{2},q_{2}^{2})}{F_{psys}(Q_{1}0)} = \frac{m_{p}^{2}}{m_{p}^{2} - q_{1}^{2} - q_{2}^{2}} \sim \frac{1}{Q^{2}}$ $\frac{F_{1}^{s}(q_{1}^{2},q_{2}^{2})}{F_{1}^{s}(0,0)} = \frac{m_{s}^{2}(3m_{s}^{2}-q_{1}^{2}-q_{2}^{2})}{3(m_{s}^{2}-q_{1}^{2}-q_{2}^{2})} - \frac{1}{2}$ $\frac{J_{z}^{s}(q_{1}^{2},q_{2}^{2})}{J_{z}^{s}(q_{1}^{2},q_{2}^{2})} = \frac{2u_{s}^{s}}{3(m_{s}^{2}-q_{1}^{2}-q_{2}^{2})^{2}} \sim \frac{1}{2} \sqrt{q} \left(\frac{1}{q} J_{1}^{s}\right)$ J.5(0,0) F.(9,2,92) =0 $= \frac{J_{1}^{A}(q_{1}^{2}q_{1}^{2})}{J_{1}^{A}(q_{2}^{2})} = \frac{J_{3}^{A}(q_{1}^{2},q_{1}^{2})}{J_{1}^{A}(q_{2}^{2})} = \frac{m_{A}^{2}}{m_{A}^{2}-q_{1}^{2}} \frac{1}{2} \frac{1}{\sqrt{2}} \frac{1$ 2



regarding the asymptotic limits

F Then keeping the axial weeksr mass in the integral of F2 (qi eq 2), one instead finds 2 Cant $\mathcal{F}_{2}^{A}\left(q_{10}^{2}\right) = \frac{3\mathcal{F}_{A}}{q^{4}} \frac{m_{A}^{2}}{x^{2}} \left(\frac{x}{1-x} + \log\left(1-x\right)\right) = \frac{m_{A}^{2}}{q^{2}} \frac{\mathcal{F}_{A}}{\mathcal{F}_{A}} \frac{\mathcal{F}_{A}}{\mathcal{F}_{A}} + \log\left(1-x\right) = \frac{m_{A}^{2}}{q^{2}} \frac{\mathcal{F}_{A}}{\mathcal{F}_{A}} + \log\left(1-x\right) =$ antim this? Find while hopped configurated Note that additional phenomenological input that could constrain FAH is to win fin ity? Scarce. We can, however, Consider these decay constants as they have been estimated using light-care sum rules (LCSRS), where, in particular, results for a = 0,3,8 are provided. To extract FA for the physical mesons, mining effects need to be taken into account, we introduce the mixing angle of via $\begin{pmatrix} f_{1} \\ f_{1} \end{pmatrix} = \begin{pmatrix} cos \partial_{A} & sin \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} f^{\circ} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} cos \partial_{A} & -sin \partial_{A} \end{pmatrix} \begin{pmatrix} f_{1} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f_{1} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f_{1} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f_{1} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f_{1} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f_{1} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f_{1} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\ast} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\circ} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\circ} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\circ} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\circ} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\circ} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\circ} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\circ} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\circ} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\circ} \end{pmatrix} = \begin{pmatrix} sin \partial_{A} & cos \partial_{A} \end{pmatrix} \begin{pmatrix} f^{\circ} \\ f^{\circ} \end{pmatrix} = \begin{pmatrix} f^{\circ} & f^{\circ} \end{pmatrix} = \begin{pmatrix} f^{\circ} & f^{\circ} \\ f^{\circ} \end{pmatrix} = \begin{pmatrix} f^{\circ} & f^{\circ} \end{pmatrix} = \begin{pmatrix} f^{\circ} & f^{\circ} \\ f^{\circ} \end{pmatrix} = \begin{pmatrix} f^{\circ} & f^{\circ} \end{pmatrix} = \begin{pmatrix} f^{\circ}$ Fron SU(3) Synnicetry, we have $Tv(Q^{2} \neq) = \hat{5}(3c, +266, +136)$ = 5 (3a, +2-76 [Cost f, - sid A f,] +37 [sid A f, + cost A. f,]) = J (30, + f, [276 cola + 13 sulf] + f! [-13 costa - 2-16 sida]) So that together with the definition of tos, we find Nalbenotica For (fi) I me. cot 2 (OA - Oo), Oo = arcs: 13. (*) For (fi) = me. cot 2 (OA - Oo), Oo = arcs: 13. (*) Mixing angle for whice Mixing angle for which two-phother coupling of fi vonsches Likewise, an empirical width for thre Q. (1760) can be extracted from SUCS) symmetry. $\overline{T}_{\partial \sigma}(a_i) = \frac{\overline{T}_{\partial \sigma}(F_i)}{3 \cos^2(\theta_{\mu} - \theta_0)} \frac{m_{\kappa_i}}{m_{f_i}}$ $= \frac{m_{f_i} \overline{f_{so}}(f_i) + m_{f_i'} \overline{f_{so}}(f_i)}{3m_{f_i} m_{f_i'}} = 2 c_i f_i k_e V$ errors alded in quedrature and a 2 Musert (*) in RHS to get to LHS. generic SU(3) uncertain





From the literature, we have 22 FA = 245 (13) MeV, 12 FA = 239 (13) NeV, 12 FA = 238 (10) MeV $M_{A}^{\circ} = 1,28(6) \text{ GeV}, M_{A}^{3} = 1,29(5) \text{ GeV},$

So that $F_{t}^{ell} = \Lambda 46.(7)(\Lambda 2)$ MeV, $F_{t}^{ell} = -\Lambda 22(\Lambda 1)(\Lambda 1)$ Her Fait = 12(5) per.

Using the dipple ansate $F_2(q_1^2, o) = F_2(q_0) \left(1 - \frac{q_1^2}{R^2}\right)^{-2}$ will git parameters F38 (3,1(4)=3,5 (6) (5) Ker 13-2 (6) (7) Ker N (fi/4') = 1, 04 (6)(5) Gev/0, 526(72)(31)Gev

We find that the effective decay constant Fall = F2(0,0) Z as also suggested in the literature - exceeds the above estimate by a factor of about 2. Furthermore, we can estrapdate the dipole fit to had

 $\begin{aligned} \mathcal{F}_{2}^{A}(q_{1}^{2},0) &= \frac{\mathcal{F}_{2}^{A}(q_{2}^{0},0) \, \mathsf{N}^{4}}{q_{4}^{4}} &= \frac{\mathcal{F}_{2}^{A}(q_{1}^{0},0) \, \mathsf{N}^{4}}{q_{4}^{4}} \\ & & & \\ \mathcal{F}_{N}^{A} &= \frac{\mathcal{F}_{2}^{A}(q_{2}^{0},0) \, \mathsf{N}^{4}}{3 \, \mathsf{m}_{N}^{3}} , & & \\ \mathcal{S}_{N} &= \frac{\mathcal{F}_{N}^{A}(q_{2}^{0},0) \, \mathsf{N}^{4}}{3 \, \mathsf{m}_{N}^{3}} , & & \\ \end{aligned}$

F=# = \$2 (26) mer, F= - 34 (12) rer,

i.e. even lower coefficients. Haverer, in ball case, there

is aly a single bin above Marer, rendering conclusions about the asymptotics highly uncertain.

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