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$B \rightarrow \bar{l} \nu_l l' \bar{l}'$ (Beube et al., 2021) 1

02.08.2021

In the following, we want to collect some calculations from $\langle 2102.10060 \rangle$ (Beube et al., 2021); see also the handwritten notes for Beube et al., 2011 and the corresponding Mathematica NB.

Recall from the Beube et al., 2011 calculation that

$$A(B^- \rightarrow \bar{l} \nu_l) = \frac{G_F V_{ub}}{\sqrt{2}} \langle \bar{l} \nu_l | \bar{l} \gamma_\mu (1-\gamma_5) V \cdot \bar{u} \gamma^\mu (1-\gamma_5) b | B^- \rangle$$

$$= \frac{G_F V_{ub}}{\sqrt{2}} \left\{ \bar{l} \epsilon_\nu^* \bar{u} \gamma_\mu (1-\gamma_5) V_\nu T^{\mu\nu}(p, q) - i e Q_l \bar{l} \epsilon_\nu^* (1-\gamma_5) V_\nu \right\}$$

$$T^{\mu\nu}(p, q) = -i \int d^4x e^{iq \cdot x} \langle 0 | T \{ j_{em}^\nu(x) [\bar{l} \gamma^\mu (1-\gamma_5) b](0) \} | B^- \rangle$$

where $p = m_B v$ is the momentum of the B meson, q is the momentum of the photon, and

$$j_{em}^\nu(x) = \sum_f Q_f \bar{q} \gamma^\nu q + Q_e \bar{l} \gamma^\nu l$$

is the "usual" convention and also, what difference would a sign change make?

is the electromagnetic current; the convention for the covariant derivative used in the above is $iD^\mu = i\partial^\mu - Q_e A^\mu$.
 \uparrow
 $= -1$ for leptons

Changing the meaning of μ and ν in the above matrix element and redefining the hadronic tensor without the factor of $(-i)$,

we can then obtain the amplitude for $B^- \rightarrow \bar{l} \nu_l l' \bar{l}'$ by appending the $\frac{e^2}{2s_W^2}$ piece as per (again with the covariant derivative as given above)

restricted to $l=l'$ in the paper already here? It is said that $l=l'$ requires additional kinematic considerations \rightarrow similar to $A \rightarrow 4\pi$ decay, there would be additional diagrams we would have to consider

$$A(B^- \rightarrow \bar{l} \nu_l l' \bar{l}') = \frac{G_F V_{ub}}{\sqrt{2}} \langle \bar{l}(p_1) \bar{\nu}_l(p_2) l'(q_1) \bar{l}'(q_2) | \bar{l} \gamma^\nu (1-\gamma_5) V_\nu \cdot \bar{u} \gamma_\nu (1-\gamma_5) b | B^-(p) \rangle$$

Non-identical lepton flavors here for now (for identical, see later; however, the amplitude ^{incl} remains the same with an additional diagram $+ \frac{-ig_W^2}{2s_W^2} Q_e \bar{l} \gamma^\nu (1-\gamma_5) V_\nu \bar{u} \gamma_\nu (1-\gamma_5) b$)

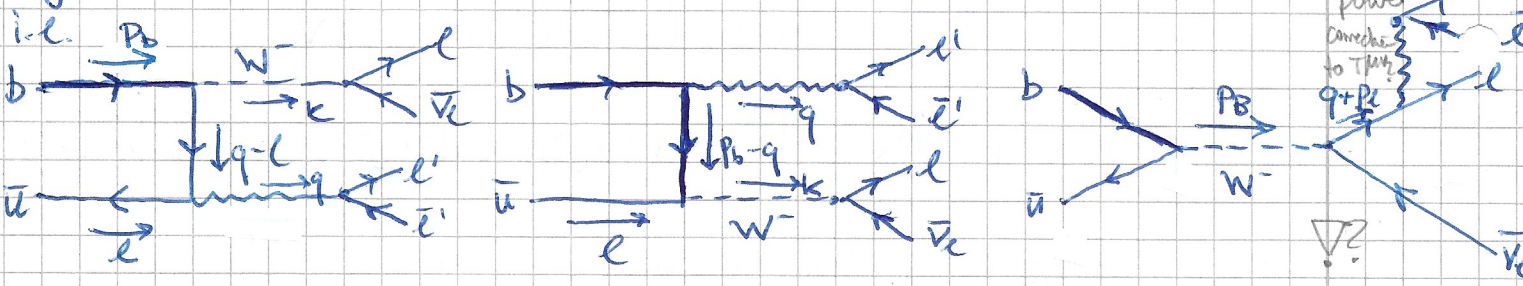
$$= \frac{G_F V_{ub}}{\sqrt{2}} \frac{ie^2}{q^2} Q_{el} [T^{\mu\nu}(p,q) + Q_e f_B g^{\mu\nu}] (\bar{u}_e \gamma_\mu v_{e'}) (\bar{u}_l \gamma_\nu (1-\gamma_5) v_l)$$

Note (04.11.2021): $q_\mu (-) = 0$ (using Dirac eq. for massless fermions; including lepton mass and rewriting the same term, with Dirac eq. yields the same). (*)

where $q = q_1 + q_2$, $p = q + k$, and $k = p_2 + p_1$ is the momentum of the virtual W Boson. Here, the hadronic tensor is given by

$$T^{\mu\nu}(p,q) = \int d^4x e^{iq \cdot x} \langle 0 | T \{ j_\mu^\dagger(x) [\bar{u}_l \gamma^\nu (1-\gamma_5) b] (0) \} | B^- \rangle,$$

which accounts for the emission of a virtual photon from the B meson constituents; the second term in the square brackets of (*) corresponds to the emission from the final-state lepton



(Note that the latter contribution constitutes a power correction relative to the $T^{\mu\nu}$ term in the kinematic region of interest.)

Using Lorentz covariance, the hadronic tensor can be decomposed into six form factors $F_i(q^2, k^2)$ of two kinematic invariants

$$T^{\mu\nu} = F_1 g^{\mu\nu} + F_2 \epsilon^{\mu\nu\alpha\beta} k_\alpha q_\beta + F_3 k^\mu q^\nu + F_4 q^\mu k^\nu + F_5 k^\mu k^\nu + F_6 q^\mu q^\nu$$

$\epsilon^{0123} = +1$

Terms with q^μ or k^ν for $\gamma^* \rightarrow l' \bar{l}'$ and $W^* \rightarrow l \bar{\nu}_l$ do not contribute to the decay amplitude for massless l' and l , as evident from the Dirac equation. However, we have to use the Ward identity

$$q_\mu T^{\mu\nu} = f_B p^\nu = f_B (k+q)^\nu \approx \int \bar{u}_l \gamma^\nu v_l, \bar{u}_l \not{k} (1-\gamma_5) v_l$$

before microlocalization this leads to $q_0 = -q_3, k = p_1 + p_2$

Why is the 2nd contr. a power correction to $T^{\mu\nu}$? $q+k$

Spectral quasi- \bar{u} has mass $l \sim \Lambda_{QCD}$ so that propagator joining W and l lines has become collinear virtuality $(q-l)^2 = 2q \cdot l \sim m_l \Lambda_{QCD}$ for first diagram and hard virtuality $(p-q)^2 \sim m_l^2$ for second diagram \rightarrow leading power corr. given by emission from light antiquark

- 1) only works if photon on-shell $q^2=0$?
- 2) why $q \cdot m_l$?
- 3) how is this suppression of 2nd diagram like one of in $T^{\mu\nu}$? Or after inner expansion for FF's

$B \rightarrow \text{LVE } \epsilon^{(1)} \epsilon^{(2)}$ (Beneke et al., 2021) 2

$$f_B (k+q)^\nu = F_1 q^\nu + F_3 (k \cdot q) q^\nu + F_4 q^2 k^\nu + F_5 (q \cdot k) k^\nu + F_6 q^2 q^\nu$$

$$\Rightarrow f_B = F_4 q^2 + F_5 (q \cdot k)$$

$$f_B = F_1 + F_3 (k \cdot q) + F_6 q^2$$

By comparing the coefficients of k^ν and q^ν . The number of independent form factors is reduced to four and eliminating F_3 and F_5 leads to

$$T^{\mu\nu} = F_1 g^{\mu\nu} + F_2 \epsilon^{\mu\nu\alpha\beta} k_\alpha q_\beta + \left(\frac{f_B - F_1 - q^2 F_6}{k \cdot q} \right) k^\mu q^\nu + F_4 q^\mu k^\nu + \left(\frac{f_B - q^2 F_4}{k \cdot q} \right) k^\mu k^\nu + F_6 q^\mu q^\nu$$

$\sim q^\mu, k^\nu$ (*)

Note that dropping $F_{4,5,6} (q^2, k^2)$ before applying the Ward identity would lead to the omission of the F_6 term in the coefficient of the $k^\mu q^\nu$ term in $T^{\mu\nu}$ and to the wrong conclusion that there are only two independent form factors $F_{1,2} (q^2, k^2)$ for massless leptons.

Note that the number of independent form factors can be associated with the number of independent polarization states of the virtual photon.

We are thus left with four form factors and a contact term after applying the Ward identity, where the q^μ, k^ν terms furthermore drop out after contracting $T^{\mu\nu}$ with the leptonic tensor and using the Dirac equation for massless fermions:

$$T^{\mu\nu} = F_1 g^{\mu\nu} + F_2 \epsilon^{\mu\nu\alpha\beta} k_\alpha q_\beta + \left(\frac{f_B - F_1 - q^2 F_6}{k \cdot q} \right) k^\mu q^\nu + \{q^\mu, k^\nu\} \text{ terms.}^*$$

Note that the contact term $\frac{f_0 k^\mu (q^\nu + k^\nu)}{k \cdot q}$ could likewise be replaced by $\frac{f_0 p^\mu (q^\nu + k^\nu)}{p \cdot q} = \frac{f_0 m_B v^\mu (q^\nu + k^\nu)}{m_B (v \cdot q)} = \frac{f_0 v^\mu (q^\nu + k^\nu)}{v \cdot q}$

(where, however, the $\{q^\mu, k^\nu\}$ terms have to be taken into account to find $q^\mu T^{\mu\nu} = f_0 (k^\nu + q^\nu)$).

We can then rewrite the hadronic tensors according to

$$\begin{aligned}
 T^{\mu\nu} &= F_1 g^{\mu\nu} + F_2 \epsilon^{\mu\nu\alpha\beta} k_\alpha q_\beta + \left(\frac{f_0 - F_1 - q^2 F_0}{k \cdot q} \right) k^\mu q^\nu + \{q^\mu, k^\nu\} \text{ terms} \\
 \stackrel{s=p \cdot q}{=} \stackrel{m_B v \cdot q}{=} & F_1 g^{\mu\nu} \frac{v \cdot q}{v \cdot q} - \frac{F_1}{v \cdot q} v^\mu q^\nu + \frac{F_1}{v \cdot q} v^\mu q^\nu + F_2 \epsilon^{\mu\nu\alpha\beta} (m_B v_\alpha - q_\alpha) q_\beta \\
 & + \left(\frac{f_0 - F_1 - q^2 F_0}{k \cdot q} \right) (m_B v^\mu - q^\mu) q^\nu + \{q^\mu, k^\nu\} \text{ terms} \\
 & = (g^{\mu\nu} (v \cdot q) - v^\mu q^\nu) \frac{F_1}{v \cdot q} + \epsilon^{\mu\nu\alpha\beta} v_\alpha q_\beta \frac{m_B F_2}{v \cdot q} \\
 & + \left(\frac{F_1}{v \cdot q} + m_B \frac{f_0 - F_1 - q^2 F_0}{k \cdot q} \right) v^\mu q^\nu + \{q^\mu, k^\nu\} \text{ terms} \\
 & = -\hat{F}_{A\perp} \frac{f_0 - F_1 - q^2 F_0}{k \cdot q} = -\hat{F}_{A\perp} - \frac{F_1}{v \cdot q} \quad \text{with } \hat{F}_{A\perp} = f_0 - F_1 + \frac{F_{A\perp} + F_1 (v \cdot q)}{m_B k \cdot q} \\
 & \quad \text{Mathematical: } \frac{f_0 - F_1 - q^2 F_0}{k \cdot q} = \frac{f_0 - F_1 - q^2 F_0}{m_B k \cdot q} + \frac{F_{A\perp} + F_1 (v \cdot q)}{m_B k \cdot q} = \frac{f_0 - F_1 + F_{A\perp} + F_1 (v \cdot q)}{m_B k \cdot q} = \frac{f_0 - F_1 + F_{A\perp}}{m_B k \cdot q} + \frac{F_1 (v \cdot q)}{m_B k \cdot q} = \frac{f_0 - F_1 + F_{A\perp}}{m_B k \cdot q} + \frac{F_1}{v \cdot q} = 0
 \end{aligned}$$

in the paper, eq. (2.3), it looks more like the decomposition was made with eq. 1) p^μ and eq. 2) contact term with $v^\mu v^\nu$ but 3) still $\{q^\mu, k^\nu\}$ terms

Now, the virtual photon emission from the final-state lepton in (*) is exactly cancelled by the redefinition

$$\hat{F}_{A\perp} = \hat{F}_{A\perp} + \frac{Q_e f_0}{v \cdot q}, \quad \hat{F}_{A\parallel} = \hat{F}_{A\parallel} - \frac{Q_e f_0}{v \cdot q},$$

since

$$\begin{aligned}
 T^{\mu\nu} &= (g^{\mu\nu} (v \cdot q) - v^\mu q^\nu) \left(\hat{F}_{A\perp} - \frac{Q_e f_0}{v \cdot q} \right) + i \epsilon^{\mu\nu\alpha\beta} v_\alpha q_\beta F_V \\
 &\quad - \left(\hat{F}_{A\parallel} + \frac{Q_e f_0}{v \cdot q} \right) v^\mu q^\nu + \{q^\mu, k^\nu\} \text{ terms} \\
 &= (g^{\mu\nu} (v \cdot q) - v^\mu q^\nu) \hat{F}_{A\perp} + i \epsilon^{\mu\nu\alpha\beta} v_\alpha q_\beta F_V - \hat{F}_{A\parallel} v^\mu q^\nu \\
 &\quad - Q_e f_0 g^{\mu\nu} + \frac{Q_e f_0}{v \cdot q} v^\mu q^\nu - \frac{Q_e f_0}{v \cdot q} v^\mu q^\nu + \{q^\mu, k^\nu\} \text{ terms.}
 \end{aligned}$$

B → $e^+e^-e^+e^-$ (Beneke et al., 2001) 3

Alternatively, we may also consider

$$T^{\mu\nu} + Q_e f_{\alpha\beta} g^{\mu\nu} = (g^{\mu\nu} (v \cdot q) - v^\mu q^\nu) \left(\hat{F}_{A_1} + \frac{Q_e f_{\alpha\beta}}{v \cdot q} \right) + i \epsilon^{\mu\nu\alpha\beta} v_\alpha q_\beta F_V - \left(\hat{F}_{A_0} - \frac{Q_e f_{\alpha\beta}}{v \cdot q} \right) v^\mu q^\nu + \{q^\mu, k^\nu\} \text{ term}$$

This is actually the more natural way, since, otherwise, one would indeed have to alter the form factors in some way!!

i.e. absorb the photon emission from the FSR into the form factors

Hence, the term in square brackets in (*) can be expressed in terms of three form factors.

27.08.2021

Introducing the light-cone vectors $n_{\pm}^\mu = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \pm 1 \end{pmatrix}$, we have

$$n_+^2 = 0 = n_-^2, \quad n_+ \cdot n_- = 2.$$

Without loss of generality, we can assume \vec{q} to point in the positive z-direction, $q = \begin{pmatrix} E_q \\ \vec{q} \end{pmatrix}$, $k = \begin{pmatrix} E_k \\ \vec{k} \end{pmatrix}$, $\vec{q} = \begin{pmatrix} 0 \\ 0 \\ |\vec{q}| \end{pmatrix}$,
 So that \vec{k} is in B rest frame

$$q^\mu = (n_+ \cdot q) \frac{n_+^\mu}{2} + (n_- \cdot q) \frac{n_-^\mu}{2}$$

(note that a similar relation holds for k^μ in the B rest frame)

Given $p^\mu = m_B v^\mu$, we find

$$v^\mu = \frac{n_+^\mu + n_-^\mu}{2}$$

in the B rest frame.

Furthermore, we find

$$q^2 = (n_+ \cdot q) (n_- \cdot q),$$

update (28.09.2021): for Stephan's notes, we might also need

$$(n_- \cdot q) = \frac{m_B^2 - k^2 + q^2 - \lambda(m_B^2, q^2, k^2)}{2m_B}$$

$$(n_+ \cdot q) = \frac{m_B^2 - k^2 + q^2 + \lambda(m_B^2, q^2, k^2)}{2m_B}$$

where $\lambda(m_B^2, q^2, k^2)$ is the Källén function.

We now introduce the transverse metric tensor as $g_{\perp}^{\mu\nu} =$

$$g_{\perp}^{\mu\nu} = g^{\mu\nu} - \frac{n_+^{\mu} n_-^{\nu} + n_-^{\mu} n_+^{\nu}}{2}$$

which fulfills

$$q_{\mu} g_{\perp}^{\mu\nu} = 0 \quad (= g_{\perp}^{\mu\nu} k_{\nu})$$

Why define the transverse metric tensor (transverse to q) and why like this?

We now rewrite

$$\begin{aligned} T^{\mu\nu} + Q_e f_3 g^{\mu\nu} &= [g^{\mu\nu} (v \cdot q) - v^{\mu} q^{\nu}] F_{A\perp} + i F_V \epsilon^{\mu\nu\alpha\beta} v_{\alpha} q_{\beta} \\ &\quad - \tilde{F}_{A\parallel} v^{\mu} q^{\nu} + \{q^{\mu} k^{\nu}\}\text{-terms} \\ &= \underbrace{\left[g^{\mu\nu} - \frac{n_+^{\mu} n_-^{\nu} + n_-^{\mu} n_+^{\nu}}{2} \right]}_{g_{\perp}^{\mu\nu}} (v \cdot q) F_{A\perp} \\ &\quad + \frac{n_+^{\mu} n_-^{\nu} + n_-^{\mu} n_+^{\nu}}{2} (v \cdot q) F_{A\perp} - F_{A\perp} v^{\mu} q^{\nu} \\ &\quad + i F_V \epsilon^{\mu\nu\alpha\beta} v_{\alpha} q_{\beta} - \tilde{F}_{A\parallel} v^{\mu} q^{\nu} + \{q^{\mu} k^{\nu}\}\text{-terms} \quad (***) \end{aligned}$$

and use that (see considerations before)

$$\begin{aligned} v^{\mu} - \frac{1}{n_+ \cdot q} q^{\mu} &= \frac{n_+^{\mu}}{2} \left(1 - \frac{n_- \cdot q}{n_+ \cdot q} \right) \\ v^{\mu} - \frac{1}{n_- \cdot q} q^{\mu} &= \frac{n_-^{\mu}}{2} \left(1 - \frac{n_+ \cdot q}{n_- \cdot q} \right) \end{aligned}$$

to express n_+ and n_- in terms of v and q , leading to (again reabsorbing some q^{μ} terms and inserting $v^{\nu} = \frac{1}{m_B} (k^{\nu} + q^{\nu})$, where $\sim k^{\nu}$ can be reabsorbed as well)

$$\begin{aligned} (***) &= \left[g^{\mu\nu} - \frac{n_+^{\mu} n_-^{\nu} + n_-^{\mu} n_+^{\nu}}{2} \right] (v \cdot q) F_{A\perp} + i F_V \epsilon^{\mu\nu\alpha\beta} v_{\alpha} q_{\beta} \\ &\quad - \left[\tilde{F}_{A\parallel} - \frac{2q^2 (m_B^2 - q^2 + k^2)}{4(m_B^2, q^2, k^2)} F_{A\perp} \right] v^{\mu} q^{\nu} + \{q^{\mu} k^{\nu}\}\text{-terms} \\ &= F_{A\parallel} \end{aligned}$$

Does not vanish for $q^2 \rightarrow 0$ but becomes $F_{A\parallel}$?

Here, $F_{A\parallel}$ arises from a longitudinally polarized virtual photon and vanishes in the real-photon limit $q^2 \rightarrow 0$.

$B \rightarrow e \bar{\nu}_e e' \bar{e}'$ (Beneke et al., 2021) 4

We thus have

$$(**) = F_{A_+} g_{+}^{\mu\nu} (\bar{v} \cdot q) + i F_V \epsilon^{\mu\nu\alpha\beta} v_\alpha q_\beta - F_{A_+} v^\mu q^\nu + \{g^{\mu\nu}, k^\nu\} \dots$$

while fulfills $q_\mu g^{\mu\nu} = 0$

$$q_\mu (**) = -F_{A_+} (\bar{v} \cdot q) q^\nu + \dots$$

31.08.2021

In order to calculate the branching ratio of the decay, we use the formula

$$d\Gamma = \frac{1}{2m_B} |M|^2 d\Phi_4(p_i, q_1, q_2, p_f, p_{\bar{f}}),$$

where

$$\begin{aligned} d\Phi_4(p_i, q_1, q_2, p_f, p_{\bar{f}}) &= d\Phi_2(q_1, q_2) d\Phi_2(p_i, p_f, p_{\bar{f}}) \frac{dq^2}{2\omega} \\ &= d\Phi_2(q_1, q_2) d\Phi_2(k, p_f, p_{\bar{f}}) d\Phi_2(p_i, k) \frac{dq^2}{2\omega} \frac{dk^2}{2\omega}, \end{aligned}$$

$$d\Phi_2 = d\Omega_{\text{cms}} \frac{2|\vec{p}_{\text{cms}}|}{32\pi^2 E_{\text{cms}}} = \frac{d\Omega_{\text{cms}}}{16\pi^2} \frac{|\vec{p}_{\text{cms}}|}{E_{\text{cms}}}$$

Recall that the amplitude is given by $\hat{=}$ ^(reabsorbed) $\text{new } T^{\mu\nu}(p, q)$ _{above}

$$A(B^- \rightarrow e \bar{\nu}_e e' \bar{e}') = \frac{G_F V_{ub}}{\sqrt{2}} \frac{ie^2}{q^2} Q_e \left[T^{\mu\nu}(p, q) + Q_e f_B g^{\mu\nu} \right] \times (\bar{u}_e \gamma_\mu \bar{v}_{e'}) (\bar{u}_{e'} \gamma_\nu (1-\gamma_5) v_B)$$

non-identical lepton flavors here, for identical ones, see later

so that

$$|A(B^- \rightarrow e \bar{\nu}_e e' \bar{e}')|^2 = \frac{G_F^2 |V_{ub}|^2}{2} \frac{e^4}{q^4} T^{\mu\nu}(p, q) T_{\mu\nu'}(p, q) \times (\bar{u}_e \gamma_\mu \bar{v}_{e'}) (\bar{u}_{e'} \gamma_\nu (1-\gamma_5) v_B) \times (\bar{u}_e \gamma_\nu (1-\gamma_5) v_B) (\bar{u}_{e'} \gamma_{\mu'} (1+\gamma_5) v_{e'})$$

$$= \frac{1}{2} |\tilde{A}|^2, \text{ i.e. without the brackets}$$

Some of the angular integrations can be carried out unambiguously, resulting in a factor of $8\pi^2$, so that (as usual, we have to calculate/obtain the kinematics in the different CMS' and then perform a Lorentz transformation to a single CMS, in this case the B-meson rest frame, see the Mathematical NB)

$$\frac{d^5 B(B^- \rightarrow \bar{\nu}_e e^+ e^- \bar{e}^-)}{d\Omega d^2 k^2 d\cos\theta_2 d\cos\theta_3 d\phi} = \frac{1}{\Gamma_B} \frac{1}{2m_B} \frac{G_F^2 |V_{ub}|^2}{2} \frac{e^4}{q^4} |\tilde{A}|^2$$

$$\alpha = \frac{e^2}{4\pi} \frac{1}{(16\pi^3)^3} \frac{|\vec{p}_B|}{m_B} \frac{|\vec{p}_{e1}|}{\sqrt{q^2}} \frac{|\vec{p}_{e1}|}{\sqrt{k^2}} \frac{1}{2\pi} \frac{1}{2\pi} 8\pi^2$$

$$\downarrow = \frac{1}{\Gamma_B} \frac{1}{2m_B} \frac{G_F^2 |V_{ub}|^2}{2} \frac{16\pi^2 \alpha^2}{q^4} \frac{8\pi^2}{(16\pi^3)^2 2\pi 2\pi}$$

$$\times \frac{|\vec{p}_B|}{m_B} \frac{|\vec{p}_{e1}|}{\sqrt{q^2}} \frac{|\vec{p}_{e1}|}{\sqrt{k^2}} |\tilde{A}|^2$$

$$\stackrel{\text{Mathematical}}{\downarrow} = \frac{G_F^2 |V_{ub}|^2 \alpha^2}{4096 \pi^4 m_B^2 \Gamma_B} \frac{\sqrt{\lambda(m_B^2, k^2, q^2)^2}}{q^4} \sqrt{1 - \frac{4m_e^2}{q^2}} \left(1 - \frac{m_e^2}{k^2}\right) \tilde{f}(q^2, k^2, q)$$

contains an additional factor $\frac{1}{m_B}$.

Note that except for the momenta, we assumed massless leptons throughout the calculation. Furthermore, we made use of the scalar products

$$p^2 = m_B^2 = (q+k)^2 = q^2 + k^2 + 2(q \cdot k) \Rightarrow q \cdot k = \frac{m_B^2 - q^2 - k^2}{2},$$

$$q^2 = (q_1 + q_2)^2 = \underbrace{q_1^2}_{=m_e^2} + \underbrace{q_2^2}_{=m_e^2} + 2(q_1 \cdot q_2) \Rightarrow q_1 \cdot q_2 = \frac{q^2 - 2m_e^2}{2},$$

$$k^2 = (p_e + p_\nu)^2 = \underbrace{p_e^2}_{=m_e^2} + \underbrace{p_\nu^2}_{=0} + 2(p_e \cdot p_\nu) \Rightarrow p_e \cdot p_\nu = \frac{k^2 - m_e^2}{2},$$

$$p^2 = 0 = (k - p_e)^2 = k^2 + m_e^2 - 2(k \cdot p_e) \Rightarrow k \cdot p_e = \frac{k^2 + m_e^2}{2},$$

$$p_e^2 = m_e^2 = (k - p_\nu)^2 = k^2 - 2(k \cdot p_\nu) \Rightarrow k \cdot p_\nu = \frac{k^2 - m_e^2}{2},$$

$$q_2^2 = m_e^2 = (q - q_1)^2 = q^2 + \underbrace{q_1^2}_{=m_e^2} - 2(q \cdot q_1) \Rightarrow q \cdot q_1 = \frac{q^2}{2}$$

$$\dots \Rightarrow q \cdot q_2 = \frac{q^2}{2}.$$

$B \rightarrow \ell \bar{\nu}_\ell \ell' \bar{\ell}'$ (Beneke et al., 2021) 5

Reabsorbing the factor of $\frac{1}{m_B^2}$ into the other prefactors, we find the result from the paper will four sign changes

$$\frac{d^5 \mathcal{B}(B^- \rightarrow \ell \bar{\nu}_\ell \ell' \bar{\ell}')}{dq^2 dk^2 d\cos\theta_\omega d\cos\theta_\delta d\phi} \stackrel{\tau_B = \frac{1}{\Gamma_B}}{=} \frac{\tau_B G_F^2 |V_{ub}|^2 \alpha^2}{2^{12} \pi^4 m_B^5} \frac{\sqrt{\lambda(m_B^2, k^2, q^2)}}{q^4} \sqrt{1 - \frac{4m_\ell^2}{q^2}} \left(1 - \frac{m_\ell^2}{k^2}\right) \times f(q^2, k^2, \omega),$$

(*) see below for k^2

$$f(q^2, k^2, \omega) = k_1 \sin^2\theta_\omega \sin^2\theta_\delta + k_2 (1 + \cos^2\theta_\omega) (1 + \cos^2\theta_\delta) + k_3 \sin^2\theta_\omega \sin^2\theta_\delta \sin^2\phi + k_4 \cos\theta_\omega (1 + \cos^2\theta_\delta) + (k_5 + k_6 \cos\theta_\omega) \sin\theta_\omega \sin\theta_\delta \cos\theta_\delta \sin\phi + (k_7 + k_8 \cos\theta_\omega) \sin\theta_\omega \sin\theta_\delta \cos\theta_\delta \cos\phi + k_9 \sin^2\theta_\omega \sin^2\theta_\delta \cos\phi \sin\phi,$$

Integrating this result over the angles, we find

$$\frac{d^2 \mathcal{B}(B^- \rightarrow \ell \bar{\nu}_\ell \ell' \bar{\ell}')}{dq^2 dk^2} = \mathcal{C} \frac{16}{9} \pi (2k_1 + 8k_2 + k_3) \frac{\tau_B G_F^2 |V_{ub}|^2 \alpha^2}{2^{12} \pi^3 m_B^5} \frac{\sqrt{\lambda(m_B^2, k^2, q^2)}}{q^2} \sqrt{1 - \frac{4m_\ell^2}{q^2}} \times \left(1 - \frac{m_\ell^2}{k^2}\right) \left[8k^2 (m_B^2 + q^2 - k^2) |F_{A\perp}|^2 + 8k^2 \chi(m_B^2, k^2, q^2) |F_V|^2 + \frac{\chi^2(m_B^2, k^2, q^2)}{q^2} |F_{A\parallel}|^2 \right],$$

in accordance with the result from the paper. The constants k_i are given by

$$k_i = \frac{1}{4} (|F_{A\perp}|^2 - |F_V|^2 + 2|F_{A\parallel}|^2),$$

$$K_2 = \frac{1}{4} (|f_{A\perp}|^2 + |f_V|^2),$$

$$K_3 = -\frac{1}{2} (|f_{A\perp}|^2 - |f_V|^2),$$

- $K_4 = \text{Re}(f_{A\perp} f_V^*)$,
- $K_5 = -\text{Im}(f_{A\parallel} f_{A\perp}^*)$,
- $K_6 = -\text{Im}(f_{A\parallel} f_V^*)$,
- $K_7 = -\text{Re}(f_{A\parallel} f_V^*)$,
- $K_8 = -\text{Re}(f_{A\parallel} f_{A\perp}^*)$,
- $K_9 = \text{Im}(f_{A\perp} f_V^*)$,

sign different from upper $\rightarrow \pi - \theta_w$
 $\rightarrow 2\pi - \phi$

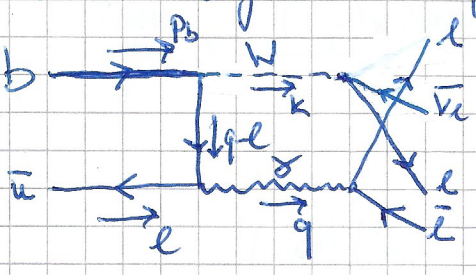
where

$$f_{A\parallel} = \chi(m_B^2, q^2, k^2) F_{A\parallel},$$

$$f_V = 2 \sqrt{k^2 q^2} \chi(m_B^2, q^2, k^2) F_V,$$

$$f_{A\perp} = 2 \sqrt{k^2 q^2} (m_B^2 - k^2 + q^2) F_{A\perp}.$$

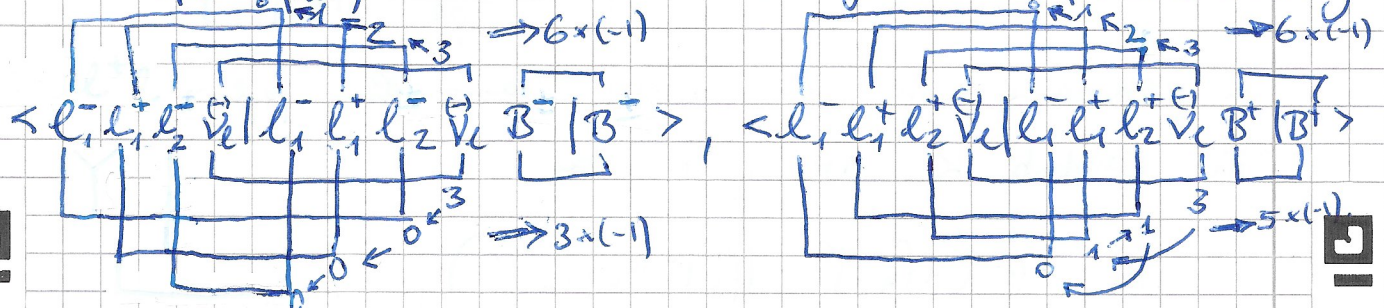
In order to work out the process for identical leptons 20.09.2021
 flavors, $l = l'$, additional kinematical considerations have to be considered, in particular an additional contribution from the interchange of the two final-state leptons, $p_c \leftrightarrow q_{112}$ (dependent on the charge of the lepton arising from the W boson). The additional diagram then, e.g., takes the form



and analogously for the other diagrams.

On the amplitude level, we have $M_{tot} = M_a - M_b$, where

$M_b = M_a(p_c \leftrightarrow q_{112})$ and the relative sign arises from considering



$B \rightarrow l \bar{\nu}_l l^{(1)} l^{(2)}$ (Beneke et al., 2021) 6

Note that for the decay rate, this results in an additional interference term between M_a and M_b , while the rates $\Gamma_{a,b} \propto |M_{a,b}|^2$ from the squares of the individual diagrams are equal. We thus find, with an additional factor of $1/2$ from the integration over the phase space due to the identical particles in the final state,

$$BR(B^- \rightarrow l \bar{\nu}_l l l) = BR(B^- \rightarrow l \bar{\nu}_l l^{(1)} l^{(2)}) + BR_{int}(B^- \rightarrow l \bar{\nu}_l l l).$$

Note that from a discussion with MÉRIL Reband, we then deduce that this factor of $1/2$ must be taken care of in $BR_{int}(B^- \rightarrow l \bar{\nu}_l l l)$ in some way in Beneke et al. 2021.

Note also that the comment "Since $\Gamma_a + \Gamma_b$ is equal to the rate for non-identical lepton flavors..." is somewhat misleading, given that this already takes into account the factor of $1/2$ from the phase-space integration.

(The paper also states that for the interference term,

$$\frac{d^2 BR_{int}(B^- \rightarrow l \bar{\nu}_l l l)}{dq^2 dk^2}, \text{ one can only obtain a numerical result.})$$